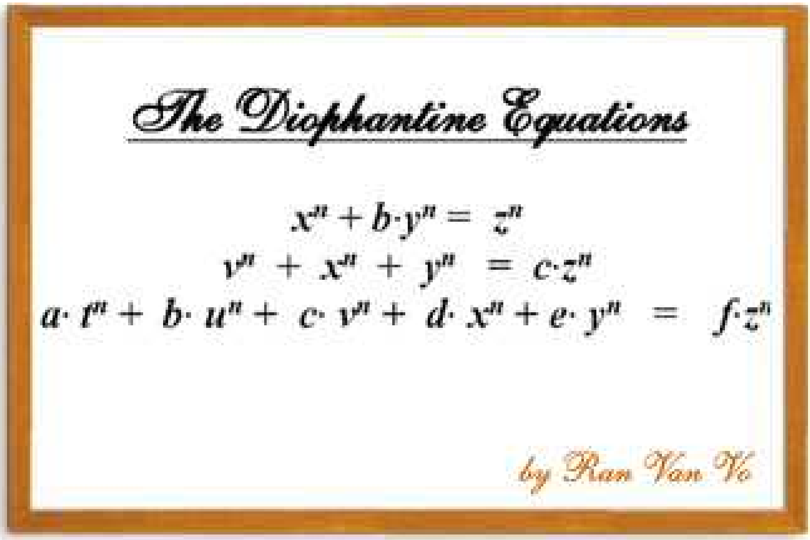
THE DIOPHANTINE EQUATIONS



First Edition

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Acknowledgments

I’m all hopeful that our common goal will be solved these Diophantine equations with the general methods, yes, we now could do by these general methods following: zeta function ζ(s) is equal to 1, and zeta function ζ(1) is equal to 0, for the Diophantine equations have exponent which is equal to 2 and zeta function ζ(s) is equal to 1, and zeta function ζ(1) is not equal to 0, for the Diophantine equations have exponent which is great than 2, or zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0, or zeta function ζ(s) is equal to c, and zeta function ζ(c) is not equal to 0, for the Diophantine equations have exponent which is great than 2. Here ‘s a good time enjoying for students ..

I welcome suggestions from Students and Readers for improvements that can be my work in the future

Wichita, KS December 15, 2003

Always sincerely

Ran Van Vo

Contents

$$V^2 + X^2 + Y^2 = Z^2$$

2) Find the value of V, X, Y, Z such that

$$V^2 + X^2 + Y^2 = Z^2$$

The Diophantine Equation $\boldsymbol{v^3 + x^3 + y^3 = z^3}$

1) Find the value of V, X, Y, Z, such that

$$V^3 + X^3 + Y^3 = Z^3$$

2) Find the value of V, X, Y, Z, such that

$$V^3 + X^3 + Y^3 = Z^3$$

3) Find the value of V, X, Y, Z such that

$$V^3 + X^3 + Y^3 = Z^3$$

4) Find the value of V, X, Y, Z such that

$$V^3 + X^3 + Y^3 = Z^3$$

5) Find the value of V, X, Y, Z such that

$$V^3 + X^3 + Y^3 = Z^3$$

6) Find the value of v, x, y, such that

$$v^3 + x^3 + y^3 = 88434^3$$

7) Find the value of v, x, y, such that

$$v^3 + x^3 + y^3 = 1581336^3$$

4) Find value a, b, c, x, y, z

$$ax^3 + by^3 = cz^3 \quad (\text{Given } a = b)$$

5) Find value a, b, c, x, y, z

$$ax^4 + by^4 = cz^4 \quad (\text{Given } a = b)$$

6) Find value a, b, c, x, y, z

$$ax^5 + by^5 = cz^5 \quad (\text{Given } a = b)$$

7) Find value a, b, c, x, y, z

$$ax^6 + by^6 = cz^6 \quad (\text{Given } a = b)$$

8) Find value x, y, z

$$524288 \cdot x^7 + 524288 \cdot y^7 = 58401973 \cdot z^7$$

9) Find value b, c, x, y, z

$$214358881 \cdot x^8 + b \cdot y^8 = c \cdot z^8$$

1)The Diophantine equation with form

$$a\cdot v^n + b\cdot x^n + c\cdot y^n = d\cdot z^n$$

2) The Diophantine equation with form

$$a\cdot u^n + b\cdot v^n + c\cdot x^n + d\cdot y^n = e\cdot z^n$$

3) The Diophantine equation with form

$$a\cdot u^n + b\cdot v^n + c\cdot x^n + d\cdot y^n = e\cdot z^n$$

4) The Diophantine equation with form

$$a\cdot t^n + b\cdot u^n + c\cdot v^n + d\cdot x^n + e\cdot y^n = f\cdot z^n$$

…

PREFACE

Dear Students and Readers

Today we have advance in science, and society has become more and more technological. It seems that we are contented with the present knowledge that we have obtained in school, however, we are still far from reaching our goal.

For example, in the past an arrowhead hit a target, we considered it a success, but today a rocket hit in an area is still not good enough, it must to land in five or ten differences areas

The same can be said for the mathematics, if we want to find a solution to an unknown, we must have an equation, for two unknowns we must have two equations and so on….

There are many equations with more than single unknown, often known as the Diophantine Equations. We know about Diophantus (200-280), which is best known for his *Arithmetica*, his work on the solution of algebraic equations and on the theory of numbers. He wrote no later than 350 AD.

For an equation with more than one unknown, then anyone solve it? It is going be very difficult! Could a calculator or computer aid us in finding the solution to an equation with more than one unknown? Of course not! However, my purpose in this book is to help students and readers could solve it.

It sometime takes a few minutes to solve it because in first book and in this book there are many new general methods that could help us in solving the equations above. I believe that there are many different ways of solving this problem besides my methods.
All of my information have been most interesting for students, I hope in this little book which is a brick, there from students step on and continue for more than research in the future, also I think it would be a good idea, may be able to help student going ahead…

The Mathematicians convention in College De France of May 2000 had come up with seven problems of the millennium. One of seven problems was the Birch and Swinnerton-Dyer Conjecture that I have solved in the first book “Fermat’s Last Theorem”. Welcome to go back my proving “Fermat’s Last Theorem” and “the Birch and Swinnerton-Dyer Conjecture” at 1st book.

CHAPTER 1.

The Diophantine Equations $(a^{2k}+ b^{2k})(c^{2k}+d^{2k}) = u^2 + v^2$

1)Product of two sums of two squares of integers has the form
$(a^2+b^2)(c^2+d^2) = u^2 + v^2$

~~~~~~~~////~~~~~~~

The product of two sums of two squares of integers a, b, c, d, we can find the sum of two squares u and v integers such as

$(a^2+b^2)(c^2+d^2) = u^2 + v^2$

We chose any value integers of a, b, c, d, now we find u and v

Example:

Given a = 17; b = 21; c = 42; d = 53

~~~~~ ***Solve*** ~~~~

Before beginning our brief survey of algebra, we touch lightly:

$(a+b)(c-d) = u^2 - v^2$

also, if the first form of the distributive law is written in reverse order, as $(a^2+b^2)(c^2+d^2) = u^2 + v^2$

$= [(a+ib)(a-ib)][(c+id)(c-id)] = [(a+ib)(c+id)][(a-ib)(c-id)] = u^2 + v^2$

$= [(ac+aid+ibc-bd)][(ac-aid-ibc-bd)] = u^2 + v^2$ ($i^2 = -1$)

$= [(ac-bd)+i(ad+bc)][(ac-bd)-i(ad+bc)] = u^2 + v^2$

We can rewrite the right side of the equation

$[(ac-bd)+i(ad+bc)][(ac-bd)-i(ad+bc)] = (u+iv)(u-iv)$

From here, we have

$u = |ac-bd|$

$iv = i(ad+bc)$ divide both sides for i we have

$v = ad+bc$

$$u = |ac-bd|$$

$$v = ad+bc$$

Substituting the value of a, b, c, and d
$u = |(17x42)-(21x53)| = 399$
$v = (17x53)+(21x42) = 1783$
We verify that
Substituting the value of a, b, c, and d into the equation

$(a^2+b^2)(c^2+d^2) = u^2 + v^2$
$(17^2+21^2)(42^2+53^2) = 399^2 + 1783^2$
product the left side of the equation:
$(17^2+21^2)(42^2+53^2) = 3338290$
Sum the right side of the equation:
$399^2 + 1783^2 = 3338290$
Solution:
$u = 399$
$v = 1783$
Similarly above
We can rewrite the left side of the equation $(a^2+b^2)(c^2+d^2) = u^2 + v^2$
$[(a+ib)(a-ib)][(c+id)(c-id)] = [(a+ib)(c-id)][(a-ib)(c+id)] = u^2 + v^2$
$[(ac-aid+ibc-ad)][(ac+aid-ibc+bd)] = u^2 + v^2$
$[(ac+bd)+i(bc-ad)][(ac+bd)-i(bc-ad)] = u^2 + v^2$
We can rewrite right side of the equation
$[(ac+bd)+i(bc-ad)][(ac+bd)-i(bc-ad)] = (u+iv)(u-iv)$
From here, we have
$u = ac+bd$
$v = |bc-ad|$
Substituting the value of a, b, c, and d
$u = (17x42)+(21x53) = 1827$
$v = |(17x53)+(21x42)| = 19$
We verify that
Substituting the value of a, b, c, and d into the equation
$(a^2+b^2)(c^2+d^2) = u^2 + v^2$

$(17^2+21^2)(42^2+53^2) = 1827^2 + 19^2$

product the left side of the equation:

$(17^2+21^2)(42^2+53^2) = 3338290$

Sum the right side of the equation:

$1827^2 + 19^2 = 3338290$

Solution 2:

u = 1827

v = 19

Conclude: The product of two sums of two squares of integers a, b, c, d, we can find the sum of two squares u and v integers such as

$(a^2+b^2)(c^2+d^2) = u^2 + v^2$ therefore

u = |ac-bd| v = ad+bc or u = ac+bd v = |bc-ad|

DIOPHANTINE EQUATION

2)Product of two sums of two numbers (fourth powers) of integers has the form

$(a^4+ b^4)(c^4+d^4) = u^2 + v^2$

We can now rewrite this equation $(a^4+ b^4)(c^4+d^4) = u^2 + v^2$ to form $(a^2+b^2)(c^2+d^2) = u^2 + v^2$ then

$(a^4+ b^4)(c^4+d^4) = [(a^2)^2+(b^2)^2][(c^2)^2+(d^2)^2] = u^2 + v^2$

get $A = a^2$; $B = b^2$; $C = c^2$ and $D = d^2$ we have

$(A^2+B^2)(C^2+D^2) = u^2 + v^2$

Similarly above

$u = \lvert AC-BD \rvert$
$v = AD+BC$
or
$u = AC+BD$
$v = \lvert BC-AD \rvert$

Example

*) Find the value of u and v which are the whole numbers such as

$(24^4+ 31^4)(105^4+47^4) = u^2 + v^2$ we rewrite to form

$(A^2+B^2)(C^2+D^2) = u^2 + v^2$

$A = a^2 = 24^2 = 576$

$B = b^2 = 31^2 = 961$

$C = c^2= 105^2 = 11025$

$D = d^2 = 47^2 = 2209$

Substituting the value of A, B, C, and D we have

$u = \lvert AC-BD \rvert = \lvert (576x11025)-(961x2209) \rvert = 4227551$

$v = AD+BC = (576x2209)+(961x11025) = 11867409$

We verify that

$(24^4+ 31^4)(105^4+47^4) = 4227551^2 + 11867409^2$

product the left side of the equation:

$(24^4+ 31^4)(105^4+47^4) = 158707583830882$

Sum the right side of the equation:

$4227551^2 + 11867409^2 = 158707583830882$

$u = 4227551$

$v = 11867409$

Similarly we have another solution

u = 8473249
v = 9322641

DIOPHANTINE EQUATION

3)Product of two sums of two numbers (6^{th} powers) of integers has the form
$$(a^6+b^6)(c^6+d^6) = u^2 + v^2$$

We can now rewrite this equation $(a^6+b^6)(c^6+d^6) = u^2 + v^2$
to form $(a^2+b^2)(c^2+d^2) = u^2 + v^2$ then
$(a^6+b^6)(c^6+d^6) = [(a^3)^2+(b^3)^2][(c^3)^2+(d^3)^2] = u^2 + v^2$
get $A = a^3$; $B = b^3$; $C = c^3$ and $D = d^3$ we have
$(A^2+B^2)(C^2+D^2) = u^2 + v^2$
Similarly above

u = |AC-BD|
v = AD+BC
or
u = AC+BD
v = |BC-AD|

Example
*) Find the value of u and v which are the whole numbers such as
$(12^6+21^6)(16^6+34^6) = u^2 + v^2$ we rewrite to form
$(A^2+B^2)(C^2+D^2) = u^2 + v^2$
$A = a^3 = 12^3 = 1728$
$B = b^3 = 21^3 = 9261$
$C = c^3 = 16^3 = 4096$
$D = d^3 = 34^3 = 39304$
Substituting the value of A, B, C, and D. We have

u = |AC-BD| = |(1728x4096)-(9261x39304)| = 356916456

v = AD+BC = (1728x39304)+(9261x4096) = 105850368

We verify that

$(12^6+21^6)(16^6+34^6) = 356916456^2 + 105850368^2$

product the left side of the equation:

$(12^6+21^6)(16^6+34^6) = 138593656969335360$

Sum the right side of the equation:

$356916456^2 + 105850368^2 = 138593656969335360$

u = 356916456

v = 105850368

Similarly we have another solution

u = 371072232

v = 29984256

DIOPHANTINE EQUATION

4) So on, product of two sums of two numbers (2k powers) of integers has the form

$\boldsymbol{(a^{2k}+b^{2k})(c^{2k}+d^{2k}) = u^2 + v^2}$

We can now rewrite this equation $(a^{2k}+b^{2k})(c^{2k}+d^{2k}) = u^2 + v^2$ to form $(a^2+b^2)(c^2+d^2) = u^2 + v^2$ then

$(a^{2k}+b^{2k})(c^{2k}+d^{2k}) = [(a^k)^2+(b^k)^2][(c^k)^2+(d^k)^2] = u^2 + v^2$

get $A = a^k$; $B = b^k$; $C = c^k$ and $D = d^k$ we have

$(A^2+B^2)(C^2+D^2) = u^2 + v^2$

Similarly above

u = |AC-BD|

v = AD+BC

or

u = AC+BD

v = |BC-AD|

CHAPTER 2

The Diophantine Equations $x^2 + y^2 = z^{2k}$

1) The Diophantine Equation $x^2 + y^2 = z^4$

There is three unknowns x, y, z. We will spend many hours for this equation by hand calculator or computer, but we have already used the formula of the Birch and Swinnerton-Dyer Conjecture as I proved at 1st book. We used the first case, zeta function ζ(s) is equal to 1, and zeta function ζ(1) is equal to 0, we want to use this formula, we must change the equation $x^2 + y^2 = z^4$

To the form Pythagoras $x^2 + y^2 = z^2$

$x^2 + y^2 = z^4 \quad = \quad x^2 + y^2 = (z^2)^2$

Put $Z = z^2$

we have $x^2 + y^2 = Z^2$

I suggested above:

$$\zeta(s) = \alpha^2 + \beta^2 = 1$$

$$\zeta(1) = \alpha^2 + \beta^2 - 1$$

$$= 1 - 1 = 0$$ (because β is rational number)

Then ζ(1) is equal to 0

Between 0 and 1 we have many values of α and β satisfying the equation :

$$\alpha^2 + \beta^2 = 1$$

$$0.6^2 + 0.8^2 = 1$$

$0.28^2 + 0.96^2 = 1$

$0.352^2 + 0.936^2 = 1$

$(20/29)^2 + (21/29)^2 = 1$

$(28/53)^2 + (45/53)^2 = 1$

$(12/37)^2 + (35/37)^2 = 1$

We have a new formula for the equation $x^2 + y^2 = z^2$ by $\zeta(1)$ is equal to zero

$x = \alpha.z$
$y = \sqrt{1 - \alpha^2} \cdot z$
$0 < \alpha < 1$

There are many solution satisfying the equation $x^2 + y^2 = z^4$

Example 1

*) Find the value of x, y, z which are the whole numbers such as

$x^2 + y^2 = z^4$.

~~~~~ solve ~~~~~
~~~~~

Change the equation $x^2+ y^2 = z^4$
To the form Pythagoras $x^2+ y^2 = z^2$
$x^2+ y^2 = z^4$ = $x^2+ y^2 = (z^2)^2$
Put $Z = z^2$
We have $x^2+ y^2 = Z^2$ we chose any value of α and β satisfying $\alpha^2 + \beta^2 = 1$
*If we choose ζ(1) is equal to 0

$\alpha = 0.6$ then $\beta = \sqrt{1 - \alpha^2} = 0.8$ therefore

$x = \alpha \cdot z = 0.6 \cdot Z$

$y = \sqrt{1 - \alpha^2} \cdot z = 0.8 \cdot Z$

*Giving1 $z = 5$ then $Z = z^2 = 25$ we have

$x = \alpha \cdot Z = 0.6 \cdot Z = 0.6 \text{ x } 25 = 15$

$y = \sqrt{1 - \alpha^2} \cdot Z = 0.8 \cdot Z = 0.8\text{x}25 = 20$

*Giving2 $z = 15$ then $Z = z^2 = 225$ we have

$x = \alpha \cdot z = 0.6 \cdot Z = 0.6\text{x}225 = 135$

$y = \sqrt{1 - \alpha^2} \cdot z = 0.8 \cdot Z = 0.8\text{x}225 = 180$

We verify that

$x^2+ y^2 = z^4$.

*1) $15^2+ 20^2 = 5^4$.= 225+400 = 625

Solution1

x = 15, y = 20, z = 5

*2) $135^2 + 180^2 = 15^4$ 18225+32400 = 50626

Solution2

x = 135, y = 180, z = 15

**If we choose ζ(1) is equal to 0

$(20/29)^2 + (21/29)^2 = 1$

$\alpha = 20/29$ then $\beta = \sqrt{1 - \alpha^2} = 21/29$ therefore

$x = \alpha \cdot Z = 20/29 \cdot Z$

$y = \sqrt{1 - \alpha^2} \cdot Z = 21/29 \cdot Z$

**Giving1 z = 116 then $Z = z^2 = 13456$ we have

$x = \alpha \cdot Z = 20/29 \cdot Z = 20/29 \times 13456 = 9280$

$y = \sqrt{1 - \alpha^2} \cdot Z = 21/29 \cdot Z = 21/29 \times 13456 = 9744$

**Giving2 z = 203 then $Z = z^2 = 41209$ we have

$x = \alpha \cdot Z = 20/29 \cdot Z = 20/29 \times 41209 = 28420$

$y = \sqrt{1 - \alpha^2} \cdot Z = 21/29 \cdot Z = 21/29 \times 41209 = 29841$

We verify that $x^2 + y^2 = z^4$.

**1) $9280^2 + 9744^2 = 116^4$ =

86118400+94945536 = 181063936

Solution1

$x = 9280$, $y = 9744$, $z = 116$

**2) $28420^2 + 29841^2 = 203^4$ =

807696400+890485281 = 1698181681

Solution2

$x = 28420$, $y = 29841$, $z = 203$

** (z must common factor 29)

~~~~~~/////~~~~~

Example 2

*) Find the value of x, y, which are the whole numbers such as

$$x^2 + y^2 = 39^4$$

~~~~~ solve ~~~~

Change the equation $x^2 + y^2 = 39^4$
To the form Pythagoras $x^2 + y^2 = z^2$
$x^2 + y^2 = 39^4$ = $x^2 + y^2 = (39^2)^2$
Put $Z = z^2$
We have $x^2 + y^2 = Z^2$
We chose any value of α and β satisfying $\alpha^2 + \beta^2 = 1$
We have $z = 39$ we choose $\zeta(1)$ is equal to 0 for the value of x and y, positive integers, therefore

$\alpha = 5/13$ then $\beta = \sqrt{1 - \alpha^2} = 12/13$, because 39 common factor 13. Using new formula:

$x = \alpha \cdot Z = 5/13 \cdot 1521 = 585$

$y = \sqrt{1 - \alpha^2} \cdot Z = 12/13 \cdot 1521 = 1404$

We verify that

$x^2 + y^2 = z^4$

$585^2 + 1404^2 = 39^4 = 342225+1971216 = 2313441$

Solution:

$x = 585$ and $y = 1404$

Example 3

*) Find the value of y, z, which are the whole numbers such as

$x^2 + y^2 = z^4$

Given $x = 1040$

~~~~~ solve ~~~~~

We rewrite this equation $x^2 + y^2 = z^4$

To the form Pythagoras $x^2 + y^2 = z^2$
~~~~~

$x^2 + y^2 = z^4 \quad = \quad x^2 + y^2 = (z^2)^2$

Put $Z = z^2$

We have $x^2 + y^2 = Z^2$ we chose any value of α and β satisfying

$\alpha^2 + \beta^2 = 1$

$\zeta(s) = (5/13)^2 + (12/13)^2 = 1$

We choose $\zeta(1)$ is equal to 0 for the value of y, z, are the whole numbers

$\alpha = 5/13$ then $\beta = \sqrt{1 - \alpha^2} = 12/13$, because, divide between two numbers

[1040/(5/13)] is equal to a square . Using new formula:

$x = \alpha \cdot Z$ then

$Z = x/\alpha = 1040/(5/13) = 2704 = 52^2$

$y = \sqrt{1 - \alpha^2} \cdot Z = 12/13 \cdot 2704 = 2496$

We verify that $x^2 + y^2 = z^4$

$1040^2 + 2496^2 = 52^4 =$

$1081600 + 6230016 = 7311616$

Solution: y = 2496 and z = 52

Example 4

*) Find the value of x, z, which are the whole numbers such as

$$x^2 + y^2 = z^4$$

Given y = 21465

~~~~~ solve ~~~~~

We rewrite this equation $x^2 + y^2 = z^4$

To the form Pythagoras $x^2 + y^2 = z^2$

$x^2 + y^2 = z^4$ = $x^2 + y^2 = (z^2)^2$

Put $Z = z^2$

We have $x^2 + y^2 = Z^2$ we chose any value of α and β satisfying $\alpha^2 + \beta^2 = 1$

$$\zeta(s) = \left(\frac{28}{53}\right)^2 + \left(\frac{45}{53}\right)^2 = 1$$

we choose ζ(1) is equal to 0 for the value of x, z, are the whole numbers

$$\alpha = \frac{28}{53}$$

$$\beta = \sqrt{1 - \alpha^2} = \frac{45}{53}$$

because, divide between two numbers 21465/(45/53) is equal to a square .

Using new formula:
~~~~~

$y = \beta \cdot Z = \sqrt{1 - \alpha^2} \cdot Z$ then

$Z = y / \sqrt{1 - \alpha^2} = 21465/(45/53) = 25281 = 159^2$

$x = \alpha \cdot Z = 28/53 \cdot 25281 = 13356$

We verify that

$x^2 + y^2 = z^4$

$13356^2 + 21465^2 = 159^4 =$

$178382736+460746225 = 639128961$

Solution: x = 21465 and z = 159

~~~~~//////~~~~~

Example 5

*) Find the value of x, y, which are the whole numbers such as

$x^2 + y^2 = 85^4$

~~~~~ solve ~~~~

Change from the equation $x^2 + y^2 = 75^4$
To the form of Pythagoras $x^2 + y^2 = z^2$
$x^2 + y^2 = 85^4 \quad = \quad x^2 + y^2 = (85^2)^2$
Put $Z = z^2$
We have $x^2 + y^2 = Z^2$

$x^2 + y^2 = 7225^2$

We find 4 values of α and β satisfying $\zeta(s) = \alpha^2 + \beta^2 = 1$ and
$\zeta(1)$ is equal to 0 for the value of x and y, positive integers, therefore

$\alpha^2 + \beta^2 = 1$

$0.6^2 + 0.8^2 = 1$

$0.28^2 + 0.96^2 = 1$

$(8/17)^2 + (15/17)^2 = 1$

$(7.2/17)^2 + (15.4/17)^2 = 1$

Already used the general method

Substituting the values of α_1

$x_1 = \alpha_1 \cdot Z = 0.6x7225 = 4335$

Substituting the values of α_2

$x_2 = \alpha_2 \cdot Z = 0.28x7225 = 2023$

Substituting the values of α_3

$x_3 = \alpha_3 \cdot Z = (8/17)x7225 = 3400$

Substituting the values of α_4

$x_4 = \alpha_4 \cdot Z = (7.2/17)x7225 = 3060$

And $y = \beta \cdot Z = \sqrt{1 - \alpha^2} \cdot Z$ then

Substituting the values of β_1

$y_1 = \beta_1 \cdot Z = \sqrt{1 - \alpha_1^2} \cdot Z = 0.8x7225 = 5780$

Substituting the values of β_2

$y_2 = \beta_2 \cdot Z = \sqrt{1 - \alpha_2^2} \cdot Z = 0.96x7225 = 6936$

Substituting the values of β_3

$y_3 = \beta_3 \cdot Z = \sqrt{1 - \alpha_3^2} \cdot Z = (15/17)x7225 = 6375$

Substituting the values of β_4

$y_4 = \beta_4 \cdot Z = \sqrt{1 - \alpha_4^2} \cdot Z = (15/17)x7225 = 6545$

Substituting the values of x, y, z, into the equation $x^2 + y^2 = z^4$

Value 1

We verify that

$x_1^2 + y_1^2 = z^4 \quad = \quad 4335^2 + 5780^2 = 85^4$

Sum the left side is equal to 52200625

And right side is equal to 52200625

Value 2

$x_2^2 + y_2^2 = z^4$ = $2023^2 + 6936^2 = 85^4$

Sum the left side is equal to 52200625

And right side is equal to 52200625

Value 3

$x_3^2 + y_3^2 = z^4$ = $3400^2 + 6375^2 = 85^4$

Sum the left side is equal to 52200625

And right side is equal to 52200625

Value 4

$x_4^2 + y_4^2 = z^4$ = $3060^2 + 6545^2 = 85^4$

Sum the left side is equal to 52200625

And right side is equal to 52200625

Solution

x_1 = 4335, y_1 = 5780

x_2 = 2023, y_2 = 6936

x_3 = 3400, y_3 = 6375

x_4 = 3060, y_4 = 6545

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DIOPHANTINE EQUATION

2) The Diophantine Equation $x^2 + y^2 = z^6$

We used the first case, zeta function ζ(s) is equal to 1, and zeta function ζ(1) is equal to 0, before using this formula, we must rewrite the equation $x^2 + y^2 = z^6$ to the form of Pythagoras
$x^2 + y^2 = z^2$
$x^2 + y^2 = z^6 \quad = \quad x^2 + y^2 = (z^3)^2$
Put $Z = z^3$
we have $x^2 + y^2 = Z^2$
we have already used the general method

$$x = \alpha.z$$

$$y = \sqrt{1 - \alpha^2}\ .z$$

$$0 < \alpha < 1$$

with first case of the Birch and Swinnerton-Dyer conjecture

$$\zeta(s) = \alpha^2 + \beta^2 = 1$$

$$\zeta(1) = \alpha^2 + \beta^2 - 1$$

$$= 1 - 1 = 0$$ (because β is rational number)

Then ζ(1) is equal to 0

Between 0 and 1 we have many values of α and β satisfying the equation :

$$\alpha^2 + \beta^2 = 1$$

$0.6^2 + 0.8^2 = 1$

$0.28^2 + 0.96^2 = 1$

$0.352^2 + 0.936^2 = 1$

$(5/13)^2 + (12/13)^2 = 1$

$(6.6/13)^2 + (11.2/13)^2 = 1$

$(8/17)^2 + (15/17)^2 = 1$

$(7.2/17)^2 + (15.4/17)^2 = 1$

$(20/29)^2 + (21/29)^2 = 1$

$(4.8/29)^2 + (28.6/29)^2 = 1$

$(12/37)^2 + (35/37)^2 = 1$

$(9/41)^2 + (40/41)^2 = 1$

$(28/53)^2 + (45/53)^2 = 1$

$(11/61)^2 + (60/61)^2 = 1$ ….

~~~~~/////~~~~

*) find the value of x, y, z, where are the whole numbers, such as
~~~~~

$x^2 + y^2 = z^6$

~~~~~ solve ~~~~

We rewrite: $x^2 + y^2 = z^6$ to the form of Pythagoras $x^2 + y^2 = z^2$
$x^2 + y^2 = z^6$ = $x^2 + y^2 = (z^3)^2$
Put $Z = z^3$
we have $x^2 + y^2 = Z^2$

first we choose α and β

Example three values of α and β below

$0.6^2 + 0.8^2 = 1$

$0.28^2 + 0.96^2 = 1$

$0.352^2 + 0.936^2 = 1$

Given any value of z then $Z = z^3$

Ex: $z = 50$ then $Z = z^3 = 50^3 = 125000$

Already used the general method

Substituting the values of α_1

$x_1 = \alpha_1 \cdot Z = 0.6x125000 = 75000$

Substituting the values of α_2

$x_2 = \alpha_2 \cdot Z = 0.28x125000 = 35000$

Substituting the values of α_3
~~~~~

$x_3 = \alpha_3 \cdot Z = 0.352x125000 = 44000$

And $y = \beta \cdot Z = \sqrt{1 - \alpha^2} \cdot Z$ then

Substituting the values of β_1

$y_1 = \beta_1 \cdot Z = \sqrt{1 - \alpha_1^2} \cdot Z = 0.8x125000 = 100000$

Substituting the values of β_2

$y_2 = \beta_2 \cdot Z = \sqrt{1 - \alpha_2^2} \cdot Z = 0.96x125000 = 120000$

Substituting the values of β_3

$y_3 = \beta_3 \cdot Z = \sqrt{1 - \alpha_3^2} \cdot Z = 0.936x125000 = 117000$

Substituting the values of x, y, z, into the equation $x^2 + y^2 = z^6$

Value 1

$x_1^2 + y_1^2 = z_1^6 \quad = \quad 75000^2 + 100000^2 = 50^6$

Sum the left side is equal to 15625000000

And right side is equal to 15625000000

Value 2

$x_2^2 + y_2^2 = z_2^6 \quad = \quad 35000^2 + 120000^2 = 50^6$

Sum the left side is equal to 15625000000

And right side is equal to 15625000000

Value 3

$x_3^2 + y_3^2 = z_3^6$ = $44000^2 + 117000^2 = 50^6$

Sum the left side is equal to 15625000000

And right side is equal to 15625000000

Solution

x_1 = 75000, y_1 = 100000

x_2 = 35000, y_2 = 120000

x_3 = 44000, y_3 = 117000

We can find more than solutions of the equation

~~~~~/////~~~~~

DIOPHANTINE EQUATION

3) The Diophantine Equation $x^2 + y^2 = z^8$

We used the first case, zeta function ζ(s) is equal to 1, and zeta function ζ(1) is equal to 0, before using this formula, we must rewrite the equation $x^2 + y^2 = z^8$ to the form of Pythagoras
$x^2 + y^2 = z^2$
$x^2 + y^2 = z^8$ = $x^2 + y^2 = (z^4)^2$
Put $Z = z^4$
we have $x^2 + y^2 = Z^2$
we have already used the general method
~~~~~

$$x = \alpha . z$$

$$y = \sqrt{1 - \alpha^2}\ .z$$

$$0 < \alpha < 1$$

with first case of the Birch and Swinnerton-Dyer conjecture

$$\zeta(s) = \alpha^2 + \beta^2 = 1$$

$$\zeta(1) = \alpha^2 + \beta^2 - 1$$

$= 1 – 1 = 0$ (because β is rational number)

Then $\zeta(1)$ is equal to 0

Between 0 and 1 we have many values of α and β satisfying the equation :

$$\alpha^2 + \beta^2 = 1$$

~~~~~/////~~~~~

*) find the value of the whole numbers of x, y, z, Such as

$\boldsymbol{x^2 + y^2 = z^8}$

~~~~~ solve ~~~~

We rewrite: $x^2+ y^2 = z^8$ to the form of Pythagoras $x^2+ y^2 = z^2$

$x^2+ y^2 = z^8 \quad = \quad x^2+ y^2 = (z^4)^2$

Put $Z = z^4$

we have $x^2 + y^2 = Z^2$

first we choose α and β

Example three values of α and β below

$(8/17)^2 + (15/17)^2 = 1$

$(20/29)^2 + (21/29)^2 = 1$

Given any value of z then $Z = z^4$

Ex: $z = 493$ then $Z = 493^4 = 59072816401$

Already used the general method

Substituting the values of α_1

$x_1 = \alpha_1 \cdot Z = (8/17)x59072816401 = 27798972424$

Substituting the values of α_2

$x_2 = \alpha_2 \cdot Z = (20/29)x59072816401 = 40739873380$

And $y = \beta \cdot Z = \sqrt{1 - \alpha^2} \cdot Z$ then

Substituting the values of β_1

$y_1 = \beta_1 \cdot Z = \sqrt{1 - \alpha_1^2} \cdot Z = (15/17)\ x59072816401 = 52123073295$

Substituting the values of β_2

$y_2 = \beta_2 \cdot Z = \sqrt{1 - \alpha_2^2} \cdot Z$ = (21/29)x59072816401 = 42776867049

Substituting the values of x, y, z, into the equation $x^2 + y^2 = z^8$

Value 1

$x_1^2 + y_1^2 = z_1^8$ = $27798972424^2 + 52123073295^2 = 493^8$

Sum the left side is equal to 3489597637546254592801

And right side is equal to 3489597637546254592801

Value 2

$x_2^2 + y_2^2 = z_2^8$ = $40739873380^2 + 42776867049^2 = 493^8$

Sum the left side is equal to 3489597637546254592801

And right side is equal to 3489597637546254592801

Solution

x_1 = 27798972424

x_2 = 40739873380 And

y_1 = 52123073295

y_2 = 42776867049

We can find more than solutions of the equation

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CHAPTER 3

The Diophantine Equations Base on Fermat-Wiles Equation

Recently I look for on internet a Diophantine equation, also mathematicians are called “a Generalized Fermat-Wiles Equation” this equation have form following:

“Fermat's Last Theorem was no more than a conjecture for over 350 years. Let n be an integer greater than 2. Fermat claimed that any integers x, y and z, not necessarily positive, for which

$$x^n + y^n = z^n$$

must consequently satisfy $x \cdot y \cdot z = 0$. *Andrew Wiles' spectacular achievement, building on the work of Kenneth Ribet and others, was to prove beyond any doubt that Fermat's conjecture is true.*

To some people, the passage of this conjecture to theoremhood is marked by sadness. They may mistakenly believe that no other interesting Diophantine equations are left to be solved. This essay is aimed at such individuals: there is a much larger class of equations, of which Fermat-Wiles is only a special case, that is well worth everyone's attention!

The equation we'll examine is

$$x^n + y^n = c \cdot z^n$$

where c is a positive integer. We wish to learn what conditions on n and c force the existence of a ***non-trivial*** *solution (x, y, z), that is,* $x \cdot y \cdot z \neq 0$. *In other words, when is the equation* x^n

+ $y^n = c \cdot z^n$ ***solvable*** *(in nonzero integers)? The case n = 1 is easy: taking x = y = c and z = 2, we conclude that non-trivial solutions always exist. The case n = 2 is somewhat more difficult. Let c' denote the square-free part of c, that is, the divisor of c which is the outcome after all factors of the form* d^2 *have been eliminated. The equation*

$$x^2 + y^2 = c \cdot z^2$$

is solvable if and only if all odd prime factors of c' are equal to 1 modulo 4. (See Hardy and Wright's[1] discussion of Waring's problem for a proof.) Here are the first several values of c for which this condition holds:..”

In this chapter, we must find the value of c for the equation

$$x^n + y^n = c \cdot z^n$$

has solutions. Similarly we have zeta function ζ(s) is equal to c and zeta function ζ(c) is equal to zero. also the value of c is the whole numbers, so much as c is fractional numbers, and conversely, if ζ(c) is not equal to 0, I will offer

I offer now, zeta function ζ(s) is equal to c and zeta function ζ(c) is equal to zero. Students can gain a thorough understanding of differential and integral zeta function ζ(s) is equal to c and zeta function ζ(c) is equal to zero. also the value of c is the whole numbers, so much as c is fractional numbers.

I've worked with several tools of mathematics also hand calculator, computer over ten years ago, This work has excellent coverage of zeta function ζ(s) is equal to c.
Give r and s which are the real numbers that can be written as

wholes numbers, or fractions, such as 3, 4, 5, 0, -7, -12… 3/2, 5/3, 7/5 -8/3 …

$\zeta(s) = r^n + s^n = c$

This is usually done by considering two cases

First case c is the whole numbers then

$\zeta(c)$ is equal to zero

Example n = 2

$\zeta(s) = r^n + s^n = c$

$\zeta(s) = 2^2 + 5^2 = 29$

1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 25, 26, ***29***, 34, 36, 37, 40, 41, 45, 49,50, 52, 53, 58, 61, 65, 68, 72, 73, 74, 80, 81, 82, 85, 89, 97, 98, 100 101, 104 106, 109, 113, 116, 117, 121, 122, 125, 128, 130, 136, 137, 144, 145, 146, 148 149, 153, 157, 162, 164, 169, 170, 173, 178, 180, 181, 185, 193, 194, 196, 197, 200, 202, 205, 208, 212, 218, 221, 225, 226, 229, 232, 233, 234, 241, 243, 245, 250, 256, 257, 260, 265, 269, 272, 274, 277, 281, 288, 289, 290, 292, 293, 296, 298, 305, 306, 313, 314, 317, 320, 324, 325, 328, 333, 337, 338, 340, 346, 349, 353, 356, 360, 361, 362, 365, 370, 373, 377, 386, 388, 389, 392, 394, 397, 400, 401, 404, 405, 409, 410, 416, 421, 424, 425, 433, 436, 441, 442, 445, 449, 450, 457, 458, 461, 464, 466, 481, 482, 484 485, 488, 493, 520, 530, 569 …

$\zeta(s) = r^n + s^n = c$

First case c is the whole numbers then

$\zeta(c)$ is equal to zero

Example n = 3

$\zeta(s) = r^3 + s^3 = c$

$\zeta(s) = 13^3 + (-12)^3 = 469$

2, 6, 7, 9, 12, 13, 15, 16, 17, 19, 20, 22, 26, 28, 30, 31, 33, 34, 35, 37, 42, 43, 48, 49, 50, 51, 53, 54, 56, 58, 61, 62, 63, 65, 67, 68, 69, 70, 71, 72, 75, 78, 79, 84, 85, 86, 87, 89, 90, 91, 92, 94, 96, 97, 98, 103, 104, 105, 106, 107, 110, 114, 115, 117, 120, 123, 124, 126, 127, 128, 130, 132, 133, 134, 136, 139, 140, 141, 142, 143, 151, 152, 153, 156, 157, 159, 160, 161, 162, 163, 164, 166, 169, 170, 171, 172, 176, 177, 178, 179, 180, 182, 183, 186, 187, 189, 193, 195, 197, 198, 201, 202, 203, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 222, 223, 224, 228, 229, 231, 233, 236, 238, 241, 243, 244, 246, 247, 249, 250, 251, 254, 258, 259, 265, 267, 269, 271, 273, 274, 275, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 289, 294, 295, 296, 301, 303, 305, 306, 308, 309, 310, 313, 314, 316, 319, 321, 322, 323, 324, 325, 330, 331, 333, 335, 337, 339, 341, 342, 344, 345, 346, 348, 349, 351, 355, 356, 357, 358, 359, 363, 366, 367, 370, 372, 373, 377, 379, 380, 382, 384, 385, 386, 387, 388, 390, 391, 393, 394, 395, 396, 397, 399, 402, 403, 405, 407, 409, 411, 413, 414, 418, 420, 421, 422, 425, 427, 428, 429, 430, 431, 432, 433, 435, 436, 438, 439, 441, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 457, 458, 459, 460, 462, 463, 465, 466, 467, 468, ***469***, 474, 477, 481, 483, 484, 485, 488, 490, 493, 494, 495, 497, 498, 499, 504, 511, 513, 520, 547, 559, 602, …

$\zeta(s) = r^n + s^n = c$

First case c is the whole numbers then

$\zeta(c)$ is equal to zero

Example n = 4

$\zeta(s) = r^4 + s^4 = c$

$\zeta(s) = 7^4 + 9^4 = 8962$

2, 17, 32, 82, 97, 162, 257, 272, 337, 512, 626, 641, 706, 881, 1250, 1297, 1312, 1377, <u>1552</u>, 1921, 2402, 2417, 2482, 2592, 2657, <u>3026</u>, 3697, 4097, 4112, 4177, 4352, 4721, 4802, 5392, 5906, 6497, 6562, 6577, 6642, 6817, 7186,7857, 8192, ***<u>8962</u>***, 10001, 10016, 10081, 10256, 10625, 10657, 11296, 12401, 13122, 14096, 14642, 14657, 14722, 14897, 15266, 15937, 16561, 17042, 18737, 20000, 20737, 20752, 20817, 20992, 21202, 21361, …

$\zeta(s) = r^n + s^n = c$

First case c is the whole numbers then

$\zeta(c)$ is equal to zero

Example n = 5

$\zeta(s) = r^5 + s^5 = c$

$\zeta(s) = 7^5 + (-5)^5 = 13682$

2, 31, 33, 64, 211, 242, 244, 275, 486, 781, 992, 1023, 1025, 1056, 1267, 2048, 2101, 2882, 3093, 3124, 3126, 3157, 3368, 4149, 4651, 6250, 6752, 7533, 7744, 7775, 7777, 7808, 8019, 8800, 9031, 10901, ***<u>13682</u>***, 15552, 15783, 15961, 16564, 16775, 16806, 16808, 16839, 17050, 17831, 19932, 24583, 24992, 26281, 29643, 31744, 32525, 32736, 32767, 32769, 32800, 33011, 33614, 33792, 35893, 40544, 40951, 42242, 49575, 51273, 55924, 58025, 58806, 59017, 59048, 59050, 59081, 59292, 60073, 61051, 62174, 65536, 66825, 67232, 68101,75856,83193, 87781, 91817, 92224, 96875, 98976, 99757, 99968, 99999,100001, 100032, 100243, 101024, 102002, 103125, 107776, 116807, 118098, 122461, 128283, 132768, 144244, 148832, 153275, 157926, 159049, 160027, 160808, 161019, 161050, 161052, 161083, 161294, 162075, 164176, 166531, 168827, 177858, 189783, 193819, …

Example n = 6

$$\zeta(s) = r^6 + s^6 = c$$

$$\zeta(s) = 8^6 + 3^6 = 262873$$

2, 65, 128, 730, 793, 1458, 4097, 4160, 4825, 8192, 15626, 15689, 16354, 19721, 31250, 46657, 47385, 50752, 62281, 93312, 117650, 117713, 118378, 121745, 133274, 164305, 235298, 262145, 262208, ***262873***, 266240, 277769, 308800, 379793, 524288, 531442, 531505, 532170, 535537, 547066, 578097, 649090, 793585, 1000001, 1000064, 1000729, 1004096, 1015625, 1046656, 1117649, 1262144, 1531441, ….

Example n = 7

$$\zeta(s) = r^7 + s^7 = c$$

$$\zeta(s) = 5^7 + 6^7 = 358061$$

2, 127, 129, 256, 2059, 2186, 2188, 2315, 4374, 14197, 16256, 16383, 16385, 16512, 18571, 32768, 61741, 75938, 77997, 78124, 78126, 78253, 80312, 94509, 156250, 201811, 263552, 277749, 279808, 279935, 279937, 280064, 282123, 296320, 358061, 543607, 559872, 745418, 807159, 821356, 823415, 823542, 823544, 823671, 825730, 839927, 901668, ….

~~~~~/////~~~~~

On a Generalized Fermat-Wiles Equation

$$x^3 + y^3 = c \cdot z^3$$

with c-values are equal to 2, 6, 7, 9, 12, 13, 15, 16, 17, 19, 20, 22, 26, 28 From internet Selmer additionally listed sample solution (x, y, z) for each of the above c-values by his calculator;

c	**x**	**y**	**z**
2	1	1	1
6	37	17	21
7	2	-1	1
9	2	1	1
12	89	19	39
13	7	2	3
15	683	397	294
~~~~~

17	18	-1	7
19	3	-2	1
20	19	1	7
22	25469	17299	9954
26	3	-1	1
28	3	1	1

But c-values are great than 28 , we do not yet know whether there is no general method general method for solving such a Generalized Fermat-Wiles Equation $x^n + y^n = c \cdot z^n$. We must to spend many hours, many days, many months or many years for the solutions of x, y, z, by calculator or computer. So far as we can use the zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to zero, for solving many solutions of Generalized Fermat-Wiles Equation $x^n + y^n = c \cdot z^n$

If we have

zeta function ζ(s) is equal to c, where c is a whole number or fractional number which is the sum of two real numbers with n^{th} powers

and zeta function ζ(c) is equal to zero. Meaning that c is rational numbers

I build this description as follows

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \to \infty$$

This fact is easy to find the value of x, y, z, when $n \to \infty$, because x, y, z, are independence with n

1) The Diophantine Equation $x^3 + y^3 = c \cdot z^3$

From new formula above, we could try to solve these equations following

*) Find value of x, y, z, where are the whole numbers such as

$x^3 + y^3 = 91 \cdot z^3$

~~~~~ solve ~~~~

Use new formula above
~~~~~

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

we have $c = 91$ then

$\zeta(s) = r^3 + s^3 = 91$

$\zeta(s) = 6^3 + (-5)^3 = 91$ and we have, too

$\zeta(s) = 3^3 + 4^3 = 91$ therefore

$x_1 = r_1 \cdot z$ and $x_2 = r_2 \cdot z$

$y_1 = s_1 \cdot z$ and $y_2 = s_2 \cdot z$

Give any value of z (z = 17) then

$x_1 = r_1 \cdot z = 6x17 = 102$

$x_2 = r_2 \cdot z = 4x17 = 68$

$y_1 = s_1 \cdot z = -5x17 = -85$

$y_2 = s_2 \cdot z = 3x17 = 51$

Substituting the values of x, y, z, into the equation

$x^3 + y^3 = c \cdot z^3$

$102^3 + -85^3 = 91 \cdot 17^3$

$1061208 - 614125 = 447083$

$68^3 + 51^3 = 91 \cdot 17^3$

$314432 + 132651 = 447083$

Solution

$x_1 = 102$, $x_2 = 68$

$y_1 = -85$, $y_2 = 51$

$z = 17$

we have infinitely values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^3 + y^3 = 91 \cdot z^3$

~~~~~/////~~~~~

*) Find value of x, y, z, which are the whole numbers such as

$x^3 + y^3 = 217 \cdot z^3$

~~~~~ solve ~~~~

We use

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

we have c = 217 (see in list n = 3 following)

2, 6, 7, 9, 12, 13, 15, 16, 17, 19, 20, 22, 26, 28, 30, 31, 33, 34, 35, 37, 42, 43, 48, 49, 50, 51, 53, 54, 56, 58, 61, 62, 63, 65, 67, 68, 69, 70, 71, 72, 75, 78, 79, 84, 85, 86, 87, 89, 90, 91, 92, 94, 96, 97, 98, 103, 104, 105, 106, 107, 110, 114, 115, 117, 120, 123, 124, 126, 127, 128, 130, 132, 133, 134, 136, 139, 140, 141, 142, 143, 151, 152, 153, 156, 157, 159, 160, 161, 162, 163, 164, 166, 169, 170, 171, 172, 176, 177, 178, 179, 180, 182, 183, 186, 187, 189, 193, 195, 197, 198, 201, 202, 203, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, ***217***, 218, 219, 222, 223, 224, 228, 229, 231, 233, 236, 238, 241, 243, 244, 246, 247, 249, 250, 251, 254, 258, 259, 265, 267, 269, 271, 273, 274, 275, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 289, 294, 295, 296, 301, 303, 305, 306, 308, 309, 310, 313, 314, 316, 319, 321, 322, 323, 324, 325, 330, 331, 333, 335, 337, 339, 341, 342, 344, 345, 346, 348, 349, 351, 355, 356, 357, 358, 359, 363, 366, 367, 370, 372, 373, 377, 379, 380, 382, 384, 385, 386, 387, 388, 390, 391, 393, 394, 395, 396, 397, 399, 402, 403, 405, 407, 409, 411, 413, 414, 418, 420, 421, 422, 425, 427, …

then

$\zeta(s) = r^3 + s^3 = c = 217$

We use hand calculator or calculation of computer for find the value of zeta function $\zeta(s) = r^3 + s^3 = c = 217$

It easy to see

$\zeta(s) = 6^3 + 1^3 = 217$ and we have, too

$\zeta(s) = 9^3 + (-8)^3 = 217$ therefore

$x_1 = r_1 \cdot z$ and $x_2 = r_2 \cdot z$

$y_1 = s_1 \cdot z$ and $y_2 = s_2 \cdot z$

Give any value of z (z = 13) then

$x_1 = r_1 \cdot z = 6x13 = 78$

$x_2 = r_2 \cdot z = 9x13 = 117$

$y_1 = s_1 \cdot z = 1x13 = 13$

$y_2 = s_2 \cdot z = -8x13 = -104$

Substituting the values of x, y, z, into the equation

$x^3 + y^3 = 217 \cdot z^3$

$78^3 + 13^3 = 217 \cdot 13^3$

474552 – 2197 = 476749

$117^3 + (-104)^3 = 217 \cdot 13^3$

1601613 – 1124864 = 476749

Solution

$x_1 = 68$

$x_2 = 117$

$y_1 = 13$

$y_2 = -104$

$z = 13$

we have many values of x, y, z, satisfying a Generalized Fermat-Wiles

Equation $x^3 + y^3 = 217 \cdot z^3$

~~~~~/////~~~~~
~~~~~

*) Find value of x, y, z, which are the whole numbers such as

$x^3 + y^3 = c \cdot z^3$

~~~~~ solve ~~~~

This equation get four unknowns x, y, z and c

We use

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

We can choose any c-value in list n = 3

… 241, 243, 244, 246, 247, 249, 250, 251, 254, 258, 259, 265, 267, 269, 271, 273, 274, 275, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 289, 294, 295, 296, 301, 303, 305, 306, 308, 309, 310, 313, 314, 316, 319, 321, 322, 323, 324, 325, 330, 331, 333, 335, 337, 339, 341, 342, 344, 345, 346, 348, 349, 351, 355, 356, 357, 358, 359, 363, 366, 367, 370, 372, 373, 377, 379, 380, 382, 384, 385, 386, 387, 388, 390, 391, 393, 394, 395, 396, 397, 399, 402,
~~~~~

403, 405, 407, 409, 411, 413, 414, 418, 420, 421, 422, 425, 427, 428, 429, 430, 431, 432, 433, 435, 436, 438, 439, 441, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 457, 458, 459, 460, 462, 463, 465, 466, 467, 468, 469, 474, 477, 481, 483, 484, 485, 488, 490, 493, 494, 495, 497, 498, 499, 504, 511, ***513***, 520, 547, 559, 602, …

example c = 513 then

$\zeta(s) = r^3 + s^3 = c = 513$

We use calculator or computer for zeta function $\zeta(s)$

$\zeta(s) = r^3 + s^3 = c = 513$

we have two sums which are equal to 513

First Sum of two whole numbers (positive or negative)

$\zeta(s) = 9^3 + (-6)^3 = 513$ and

Second Sum of two fractional numbers

$\zeta(s) = (9/2)^3 + (15/2)^3 = 513$

Now, we have already used the general method for a Generalized Fermat-Wiles Equation $x^3 + y^3 = c \cdot z^3$ therefore

$x_1 = r_1 \cdot z$

$y_1 = s_1 \cdot z$

Give any value of z (z = 72) then

$x_1 = r_1 \cdot z = 9x72 = 648$

$y_1 = s_1 \cdot z = (-6)x72 = -432$

Substituting the values of x, y, z, and c, into the equation

$x^3 + y^3 = c \cdot z^5$

$648^3 + (-432)^3 = 513 \cdot 72^3$

Sum the left side of the equation

$648^3 + (-432)^5 = 191476224$

Product the right side of the equation

$513 \cdot 72^3 = 191476224$

Second value of x, y, (z = 72) then

$x_2 = r_2 \cdot z = (9/2)x72 = 324$

$y_2 = s_2 \cdot z = (15/2)x72 = 540$

Substituting the values of x, y, z, and c, into the equation

$x^3 + y^3 = c \cdot z^3$

$324^3 + 540^3 = 513 \cdot 72^3$

Sum the left side of the equation

$324^3 + 540^3 = 191476224$

Product the right side of the equation

$513 \cdot 72^3 \quad = \quad 191476224$

Solution $c = 513$

$x_1 = 648$

$y_1 = -432$

$x_2 = 324$

$y_2 = 540$

$z = 72$

we have many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^3 + y^3 = c \cdot z^3$

~~~~~/////~~~~~

*) Find value of y, z, and c which are the whole numbers such as

$x^3 + y^3 = c \cdot z^3$

Give $x = 91$

~~~~~ solve ~~~~

There is three unknowns, if we have zeta function ζ(s) is equal to c and ζ(c) is equal to zero, it mean that c is a whole number or fractional number we use the general method for a Generalized Fermat-Wiles Equation

$x^3 + y^3 = c \cdot z^3$

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

$\zeta(s) = r^3 + s^3 = c$ therefore

$x = r \cdot z = 91$ then

$z = x/r = 91/r$ z is the whole number then $r = 7$ and $r = 91$

<u>*First casec r = 7*</u>

$z = 91/7 = 13$

$y = s \cdot z$, give any value of s (s = 8) we have

$y = s \cdot z = 8x13 = 104$

and $\zeta(s) = r^3 + s^3 = c = 7^3 + 8^3$

$c = 343+512 = 855$

Substituting the values of x, y, z, and c into the equation

$x^3 + y^3 = c \cdot z^3$

$91^3 + 104^3 = 855 \cdot 13^3$

753571 + 1124864 = 1878435

Second case r = 91

z = x/r = 91/91 = 1

y = s·z , give any value of s (s = 12) we have

y = s·z = 12x1 = 12

and $\zeta(s) = r^3 + s^3 = c = 91^3 + 12^3$

c = 753571+ 1728 = 755299

Substituting the values of x, y, z, and c into the equation

$x^3 + y^3 = c \cdot z^3$

$91^3 + 12^3 = 755299 \cdot 1^3$

753571+ 1728 = 755299

Solution1 c = 855

y_1 = 104

z_1 = 13

Solution 2 c = 755299

y_2 = 12

$z_2 \quad = \quad 1$

we have many values of y, z, and c satisfying a Generalized Fermat-Wiles Equation $91^3 + y^3 = c \cdot z^3$

~~~~~/////~~~~~

*) Find value of x, z, and c which are the whole numbers such as

$x^3 + y^3 = c \cdot z^3$

Give $y = 343$

~~~~~ solve ~~~~

There is three unknowns, if we have zeta function ζ(s) is equal to c and ζ(c) is equal to zero, it mean that c is positive integer, we use the general method for a Generalized Fermat-Wiles Equation $x^3 + y^3 = c \cdot z^3$

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \rightarrow \infty$$

$\zeta(s) = r^3 + s^3 = c$ and

$y = s \cdot z = 343$ then

$z = y/s = 343/s$,

We want z which is the whole number then $s = 7$, $s = 49$, and $s = 343$

<u>*First case s = 7*</u>

$z = y/s = 343/7 = 49$

$x = r \cdot z$, give any value of r (r = 14) we have

$x = r \cdot z = 14x49 = 686$

and $\zeta(s) = r^3 + s^3 = c = 7^3 + 14^3$

$c = 343+2744 = 3087$

Substituting the values of x, y, z, and c into the equation

$x^3 + y^3 = c \cdot z^3$

$686^3 + 343^3 = 3087 \cdot 49^3$

$322828856 + 40353607 = 363182463$

<u>*Second case s = 49*</u>

$z = y/s = 343/49 = 7$

$x = r \cdot z$, give any value of r (r = 9) we have

$x = r \cdot z = 9x7 = 63$

and $\zeta(s) = r^3 + s^3 = c = 9^3 + 49^3$

$c = 117649+729 = 118378$

Substituting the values of x, y, z, and c into the equation

$x^3 + y^3 = c \cdot z^3$

$63^3 + 343^3 = 118378 \cdot 7^3$

Third case s = 343.

Similarly we have

$z = y/s = 343/343 = 1$

$x = r \cdot z$, give any value of r (r = 15) we have

$x = r \cdot z = 15x1 = 15$

and $\zeta(s) = r^3 + s^3 = c = 15^3 + 343^3$

$c = 3375+ 40353607 = 40356982$

Substituting the values of x, y, z, and c into the equation

$x^3 + y^3 = c \cdot z^3$

$15^3 + 343^3 = 40356982 \cdot 1^3$

$3375+ 40353607 = 40356982$

Solution1 c = 3087

$x_1 = 686,\quad z_1 = 49$

Solution 2 c = 118378

$x_2 = 63,\quad z_2 = 7$

Solution 2 c = 40356982

$x_3 = 15$

$z_3 = 1$

we have many values of y, z, and c satisfying a Generalized Fermat-Wiles Equation $x^3 + y^3 = c \cdot z^3$ (y = 343)

~~~~~/////~~~~~

2) The Diophantine Equation $x^4 + y^4 = c \cdot z^4$

*) Find value of x, y, z, which are the whole numbers such as

$x^4 + y^4 = 1552 \cdot z^4$

~~~~~ solve ~~~~

We use the formula

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

c = 1552 (see in list n = 4 above)

2, 17, 32, 82, 97, 162, 257, 272, 337, 512, 626, 641, 706, 881, 1250, 1297, 1312, 1377, ***1552***, 1921, 2402, 2417, 2482, 2592, 2657, 3026, 3697, 4097, 4112, 4177, 4352, 4721, 4802, 5392, 5906, 6497, 6562, 6577, 6642, 6817, 7186,7857, 8192, 8962, 10001, 10016, 10081, 10256, 10625, 10657, 11296, 12401, 13122, 14096, 14642, 14657, 14722, 14897, 15266, 15937,….

then

$\zeta(s) = r^4 + s^4 = c = 1552$

We use calculator or computer for zeta function ζ(s)

$\zeta(s) = r^4 + s^4 = c = 1552$

we have

$\zeta(s) = 6^4 + 4^4 = 1552$ and

we have already used the general method for a Generalized Fermat-Wiles Equation $x^4 + y^4 = c \cdot z^4$ therefore

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 21) then

$x = r \cdot z = 6x21 = 126$

$y = s \cdot z = 4x21 = 84$

Substituting the values of x, y, z, into the equation

$x^4 + y^4 = 1552 \cdot z^4$

$126^4 + 84^4 = 1552 \cdot 21^4$

252047376 – 4978136 = 301834512

Give another value of z (z = 52) then

$x = r \cdot z = 6x52 = 312$

$y = s \cdot z = 4x52 = 208$

Substituting the values of x, y, z, into the equation

$x^4 + y^4 = 1552 \cdot z^4$

$312^4 + 208^4 = 1552 \cdot 52^4$

9475854336 – 1871773696 = 11347628032

Solution 1

x = 126

y = 84

z = 21

Solution 2

x = 312

y = 208

z = 52

we have many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^4 + y^4 = 1552 \cdot z^4$

~~~~~/////~~~~~

*) Find value of x, y, z, which are the whole numbers such as

$x^4 + y^4 = 3026 \cdot z^4$

~~~~~ solve ~~~~

c = 3026 (see in list n = 4 above)

2, 17, 32, 82, 97, 162, 257, 272, 337, 512, 626, 641, 706, 881, 1250, 1297, 1312, 1377, 1552, 1921, 2402, 2417, 2482, 2592, 2657, ***3026***, 3697, 4097, 4112, 4177, 4352, 4721, 4802, 5392, 5906, 6497, 6562, 6577, 6642, 6817, 7186,7857, 8192, 8962, 10001, 10016, 10081, 10256, 10625, 10657, 11296, 12401, 13122, 14096, 14642, 14657, 14722, 14897, 15266, 15937,…

then

$\zeta(s) = r^4 + s^4 = c = 3026$

We use calculator or computer for zeta function $\zeta(s)$

$\zeta(s) = r^4 + s^4 = c = 3026$

we have

$\zeta(s) = 7^4 + 5^4 = 3026$ and

we have already used the general method for a Generalized Fermat-Wiles Equation $x^4 + y^4 = c \cdot z^4$ therefore

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 27) then

$x = r \cdot z = 7x27 = 189$

$y = s \cdot z = 5x27 = 135$

Substituting the values of x, y, z, into the equation

$x^4 + y^4 = 3026 \cdot z^4$

$189^4 + 135^4 = 3026 \cdot 27^4$

1275989841 – 332150625 = 1608140466

Give another value of z (z = 32) then

$x = r \cdot z = 7x32 = 224$

$y = s \cdot z = 5x32 = 160$

Substituting the values of x, y, z, into the equation

$x^4 + y^4 = 3026 \cdot z^4$

$224^4 + 160^4 = 3026 \cdot 32^4$

2517630976 – 655360000 = 3172990976

Solution 1

x = 189

y = 135

z = 27

Solution 2

x = 224

y = 160

z = 32

we have many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^4 + y^4 = 3026 \cdot z^4$

~~~~~~/////~~~~~~
~~~~~~

3) The Diophantine Equation $x^5 + y^5 = c \cdot z^5$

*) Find value of x, y, z, which are the whole numbers such as

$x^5 + y^5 = c \cdot z^5$

~~~~~ solve ~~~~

This equation get four unknowns x, y, z and c, also we use

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

we can choose any c-value in list n = 5 above,

2, 31, 33, 64, 211, 242, 244, 275, 486, 781, 992, 1023, 1025, 1056, 1267, 2048, 2101, 2882, 3093, 3124, 3126, 3157, 3368, 4149, 4651, 6250, 6752, 7533, 7744, 7775, 7777, 7808, ***8019***, 8800, 9031, 10901, 13682, 15552, 15783, 15961, 16564, 16775, 16806, 16808, 16839, 17050, 17831, 19932, 24583, 24992, 26281, 29643, 31744, 32525, 32736, 32767, 32769, 32800, 33011, 33614, 33792, 35893, 40544, 40951, 42242, 49575, 51273, 55924, 58025, 58806, 59017, 59048, 59050, 59081, 59292, 60073, 61051, 62174, 65536,
~~~~~

66825, 67232, 68101,75856,83193, 87781, 91817, 92224, 96875, 98976, 99757, 99968, 99999,100001, 100032, 100243, 101024, 102002, 103125, 107776, 116807, 118098, 122461, 128283, 132768, 144244, 148832, 153275, 157926, 159049, 160027, 160808, 161019, 161050, 161052, 161083, 161294, 162075, 164176, 166531, 168827, 177858, 189783, 193819, …

example c = 8019 then

$\zeta(s) = r^5 + s^5 = c = 8019$

We use calculator or computer for zeta function $\zeta(s)$

$\zeta(s) = r^5 + s^5 = c = 8019$

we have

$\zeta(s) = 6^5 + 3^5 = 8019$

we have already used the general method for a Generalized Fermat-Wiles Equation $x^5 + y^5 = c \cdot z^5$ therefore

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 217) then

$x = r \cdot z = 6x217 = 1302$

$y = s \cdot z = 3x217 = 651$

Substituting the values of x, y, z, and c, into the equation

$x^5 \; + \; y^5 \; = \; c \cdot z^5$

$1302^5 \; + \; 651^5 \; = \; 8019 \cdot 217^5$

Sum the left side of the equation

$1302^5 \; + \; 651^5 \; = \; 3858503359532283$

Product the right side of the equation

$8019 \cdot 217^5 \; = \; 3858503359532283$

Give another value of z (z = 311) then

$x \; = \; r \cdot z \; = \; 6x311 \; = \; 1866$

$y \; = \; s \cdot z \; = \; 3x311 \; = \; 933$

Substituting the values of x, y, z, and c, into the equation

$x^5 \; + \; y^5 \; = \; c \cdot z^5$

$1866^5 \; + \; 933^5 \; = \; 8019 \cdot 311^5$

Sum the left side of the equation

$1866^5 \; + \; 933^5 \; = \; 23330398590836469$

Product the right side of the equation

$8019 \cdot 311^5 \; = \; 23330398590836469$

Solution 1 c = 8019

x = 1302

y = 651

z = 217

Solution 2

x = 1866

y = 933

z = 311

There is many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^5 + y^5 = c \cdot z^5$

~~~~~/////~~~~~

*) Find value of x, y, z, which are the whole numbers such as

$x^5 + y^5 = c \cdot z^5$

~~~~~ solve ~~~~

This equation get four unknowns x, y, z and c

we can choose any c-value in list n = 5 above,

2, 31, 33, 64, 211, 242, 244, 275, 486, 781, 992, 1023, 1025, 1056, 1267, 2048, 2101, 2882, 3093, 3124, 3126, 3157, 3368, 4149, 4651, 6250, 6752, 7533, 7744, 7775, 7777, 7808, 8019, 8800, 9031, 10901, 13682, 15552, 15783, 15961, 16564, 16775, 16806, 16808, <u>***16839***</u>, 17050, 17831, 19932, 24583, 24992, 26281, 29643, 31744, 32525, 32736, 32767, 32769, 32800, 33011, 33614, 33792, 35893, 40544, 40951, 42242, 49575, 51273, 55924, 58025, 58806, 59017, 59048, 59050, 59081, 59292, 60073, 61051, 62174, 65536, 66825, 67232, 68101,75856,83193, 87781, 91817, 92224, 96875, 98976, 99757, 99968, 99999,100001, 100032, 100243, 101024, 102002, 103125, 107776, 116807, 118098, 122461, 128283, 132768, 144244, 148832, 153275, 157926, 159049, 160027, 160808, 161019, 161050, 161052, 161083, 161294, 162075, 164176, 166531, 168827, 177858, 189783, 193819, …

example c = 16839 then

$\zeta(s) = r^5 + s^5 = c = 16839$

We use calculator or computer for zeta function ζ(s)

$\zeta(s) = r^5 + s^5 = c = 16839$

$\zeta(s) = 7^5 + 2^5 = 16839$

Use the general method for a Generalized Fermat-Wiles Equation

$x^5 + y^5 = c \cdot z^5$ we have

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 372) then

$x = r \cdot z = 7 \times 372 = 2604$

$y = s \cdot z = 2 \times 372 = 744$

Substituting the values of x, y, z, and c, into the equation

$x^5 + y^5 = c \cdot z^5$

$2604^5 + 744^5 = 16839 \cdot 372^5$

Sum the left side of the equation

$2604^5 + 744^5 = 11995849165458124 8$

Power and product the right side of the equation

$16839 \cdot 372^5 = 119958491654581248$

Give another value of z (z = 401) then

$x = r \cdot z = 7x401 = 2807$

$y = s \cdot z = 2x401 = 802$

Substituting the values of x, y, z, and c, into the equation

$x^5 + y^5 = c \cdot z^5$

$2807^5 + 802^5 = 16839 \cdot 401^5$

Sum the left side of the equation

$2807^5 + 802^5 = 174597555936094839$

Power and product the right side of the equation

$16839 \cdot 401^5 = 174597555936094839$

Solution 1 c = 16839

x = 2604

y = 744

z = 372

Solution 2

x = 2807

y = 802

z = 401

There is many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^5 + y^5 = c \cdot z^5$

~~~~~/////~~~~~

4) The Diophantine Equation $x^6 + y^6 = c \cdot z^6$

*) Find value of x, y, z, which are the whole numbers such as

$x^6 + y^6 = c \cdot z^6$

~~~~~ solve ~~~~

This equation get four unknowns x, y, z and c

We use

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

we can choose any c-value in list n = 6 , example c = 262873

$\zeta(s) = 8^6 + 3^6 = 262873$

2, 65, 128, 730, 793, 1458, 4097, 4160, 4825, 8192, 15626, 15689, 16354, 19721, 31250, 46657, 47385, 50752, 62281, 93312, 117650, 117713, 118378, 121745, 133274, 164305, 235298, 262145, 262208, ***262873***, 266240, 277769, 308800, 379793, 524288, 531442, 531505, 532170, 535537, 547066, 578097, 649090, 793585, 1000001, 1000064, 1000729, 1004096, 1015625, 1046656, 1117649, 1262144, 1531441, ….

$\zeta(s) = r^6 + s^6 = c = 262873$

We use calculator or computer for zeta function ζ(s)

$\zeta(s) = r^6 + s^6 = c = 262873$

$$\zeta(s) = 8^6 + 3^6 = 262873$$

Use the general method for a Generalized Fermat-Wiles Equation

$x^6 + y^6 = c \cdot z^6$ we have

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 372) then

$x = r \cdot z = 8x721 = 5768$

$y = s \cdot z = 3x721 = 2163$

Substituting the values of x, y, z, and c, into the equation

$$x^6 + y^6 = c \cdot z^6$$

$$5768^6 + 2163^6 = 262873 \cdot 721^6$$

Sum the left side of the equation

$$5768^6 + 2163^6 = 36928151580384943202233$$

Power and product the right side of the equation

$$262873 \cdot 721^6 = 36928151580384943202233$$

Give another value of z (z = 512) then

$x = r \cdot z = 8x512 = 4096$

$y = s \cdot z = 3x512 = 1536$

Substituting the values of x, y, z, and c, into the equation

$x^6 + y^6 = c \cdot z^6$

$4096^6 + 1536^6 = 262873 \cdot 512^6$

Sum the left side of the equation

$4096^6 + 1536^6 = 4735498979383057580032$

Power and product the right side of the equation

$262873 \cdot 512^6 = 4735498979383057580032$

Solution 1 c = 262873

x = 5768

y = 2163

z = 721

Solution 2

x = 4096

y = 1536

z = 512

There is many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^6 + y^6 = c \cdot z^6$

~~~~~/////~~~~~
~~~~~

5) The Diophantine Equation $x^7 + y^7 = c \cdot z^7$

*) Find value of x, y, z, which are the whole numbers such as

$x^7 + y^7 = c \cdot z^7$

~~~~~ solve ~~~~

This equation get four unknowns x, y, z and c

We use

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \to \infty$$

we can choose any c-value in list n = 7

Example c = 358061
~~~~~

$\zeta(s) = r^7 + s^7 = c$

$\zeta(s) = 5^7 + 6^7 = 358061$

2, 127, 129, 256, 2059, 2186, 2188, 2315, 4374, 14197, 16256, 16383, 16385, 16512, 18571, 32768, 61741, 75938, 77997, 78124, 78126, 78253, 80312, 94509, 156250, 201811, 263552, 277749, 279808, 279935, 279937, 280064, 282123, 296320, ***358061***, 543607, 559872, 745418, 807159, 821356, 823415, 823542, 823544, 823671, 825730, 839927, 901668, ….

$\zeta(s) = r^7 + s^7 = c = 358061$

We use calculator or computer for zeta function ζ(s)

$\zeta(s) = 5^7 + 6^7 = 358061$

Use the general method for a Generalized Fermat-Wiles Equation

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

Give any value of z (z = 973) then

$x = r \cdot z = 5 \cdot 973 = 4865$

$y = s \cdot z = 6 \cdot 973 = 5838$

Substituting the values of x, y, z, and c, into the equation

$$x^7 + y^7 = c \cdot z^7$$

$4865^7 + 5838^7 = 358061 \cdot 973^7$

Sum the left side of the equation

$4865^7 + 5838^7 = 29562890999983488587583617$

Power and product the right side of the equation

$358061 \cdot 973^7 = 29562890999983488587583617$

Give another value of z (z = 633) then

$x = r \cdot z = 5 \cdot 633 = 3165$

$y = s \cdot z = 6 \cdot 633 = 3798$

Substituting the values of x, y, z, and c, into the equation

$x^7 + y^7 = c \cdot z^7$

$3165^7 + 3798^7 = 358061 \cdot 633^7$

Sum the left side of the equation

$3165^7 + 3798^7 = 14580854974411639428725397$

Power and product the right side of the equation

$358061 \cdot 633^7 = 14580854974411639428725397$

Solution 1 $c = 358061$

$x = 4865$

$y = 5838$

$z = 973$

Solution 2

$x = 3165$

$y = 3798$

$z = 633$

There is many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^7 + y^7 = c \cdot z^7$

~~~~~/////~~~~~

6) The Diophantine Equation $x^{13} + y^{13} = c \cdot z^{13}$

*) Find value of x, y, z, which are the whole numbers such as

$x^{13} + y^{13} = c \cdot z^{13}$

~~~~~ solve ~~~~

This equation get four unknowns x, y, z and c

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

The list of zeta function $\zeta(s) = r^n + s^n = c$ (n = 13) following

$\zeta(s) = r^{13} + s^{13}$

2, 8191, 8193, <u>1586131</u>, 1594322, 1594324, 1602515, 65514541, 67100672, 67108863, 67108865, 67117056, 68703187, 1153594261,..

We choose any c-value above of zeta function ζ(s), example ζ(s) = 1586131 therefore

$\zeta(s) = r^{13} + s^{13} = c = 1586131$ then

$\zeta(s) = 3^{13} + (-2)^{13} = 1586131$

Use the general method for a Generalized Fermat-Wiles Equation

$x^{13} + y^{13} = c \cdot z^{13}$

$x^{13} + y^{13} = 1586131 \cdot z^{13}$ therefore

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 23) then

$x = r \cdot z = 3x23 = 69$

$y = s \cdot z = -2x23 = -46$

Substituting the values of x, y, z, and c, into the equation

$x^{13} + y^{13} = c \cdot z^{13}$

$69^{13} + (-46)^{13} = 1586131 \cdot 23^{13}$

Sum the left side of the equation

$69^{13} + (-46)^{13} = 7994676987946509466651 73$

Power and product the right side of the equation

$1586131 \cdot 23^{13} = 7994676987946509466651 73$

Give another value of z (z = 37) then

$x = r \cdot z = 3x37 = 111$

$y = s \cdot z = -2x37 = -74$

Substituting the values of x, y, z, and c, into the equation

$x^{13} + y^{13} = c \cdot z^{13}$

$111^{13} + -74^{13} = 1586131 \cdot 37^{13}$

Sum the left side of the equation

$111^{13} + -74^{13} = 386332697175077257010649007$

Power and product the right side of the equation

$1586131 \cdot 37^{13} = 386332697175077257010649007$

Solution 1 c = 1586131

x = 69

y = -46

z = 23

Solution 2

x = 111

y = -74

z = 37

There is many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^{13} + y^{13} = c \cdot z^{13}$

~~~~~/////~~~~~

*) Find value of x, y, z, which are the whole numbers such as

$x^{13} + y^{13} = c \cdot z^{13}$

~~~~~ solve ~~~~

This equation get four unknowns x, y, z and c

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

The list of zeta function $\zeta(s) = r^n + s^n = c$ (n = 13) following

$\zeta(s) = r^{13} + s^{13}$

2, 8191, 8193, 1586131, 1594322, 1594324, 1602515, 65514541, 67100672, 67108863, 67108865, 67117056, 68703187, 1153594261,..

We choose any c-value above of zeta function $\zeta(s)$, example $\zeta(s)$ = 65514541 therefore

$\zeta(s) = r^{13} + s^{13} = c = 65514541$

Use calculator we have

$\zeta(s) = 4^{13} + (-3)^{13} = 65514541$

Use the general method for a Generalized Fermat-Wiles Equation

$x^{13} + y^{13} = c \cdot z^{13}$

$x^{13} + y^{13} = 65514541 \cdot z^{13}$ therefore

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 41) then

$x = r \cdot z = 4x41 = 164$

$y = s \cdot z = -3x41 = -123$

Substituting the values of x, y, z, and c, into the equation

$x^{13} + y^{13} = c \cdot z^{13}$

$164^{13} + (-123)^{13} = 65514541 \cdot 41^{13}$

Sum the left side of the equation

$164^{13} + (-123)^{13} = 6060770512584415533709456661$

Power and product the right side of the equation

$65514541 \cdot 41^{13} = 6060770512584415533709456661$

Give another value of z (z = 19) then

$x = r \cdot z = 4x19 = 76$

$y = s \cdot z = -3x19 = -57$

Substituting the values of x, y, z, and c, into the equation

$x^{13} + y^{13} = c \cdot z^{13}$

$76^{13} + -57^{13} = 65514541 \cdot 19^{13}$

Sum the left side of the equation

$76^{13} + -57^{13} = 275508190921036204439 4919$

Power and product the right side of the equation

$65514541 \cdot 19^{13} = 2755081909210362044394919$

Solution 1 c = 65514541

x = 164

y = -123

z = 41

Solution 2

x = 76

y = -57

z = 19

we have many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^{13} + y^{13} = c \cdot z^{13}$

~~~~~/////~~~~~
~~~~~

7) The Diophantine Equation $x^{14} + y^{14} = c \cdot z^{14}$

*) Find value of x, y, z, which are the whole numbers such as

$x^{14} + y^{14} = c \cdot z^{14}$

~~~~~ solve ~~~~

This equation get four unknowns x, y, z and c

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

The list of zeta function $\zeta(s) = r^n + s^n = c$ (n = 14) following

$\zeta(s) = r^{14} + s^{14}$

2, 16385, 32768, 4782970, 4799353, 9565938, 268435457, 268451840, 273218425, 536870912, 6103515626, 6103532009, <u>6108298594</u>, 6371951081, 12207031250, ….

We choose any c-value above of zeta function ζ(s), example ζ(s) = 6108298594 therefore
~~~~~

$\zeta(s) = r^{14} + s^{14} = c = 6108298594$

use computer or calculator for solution of r and s

$\zeta(s) = 5^{14} + 3^{14} = 6108298594$

Use the general method for a Generalized Fermat-Wiles Equation

$x^{14} + y^{14} = c \cdot z^{14}$

$x^{14} + y^{14} = 6108298594 \cdot z^{14}$ therefore

$x = r \cdot z$ and $y = s \cdot z$

Give any value of z (z = 57) then

$x = r \cdot z = 5x57 = 285$

$y = s \cdot z = 3x57 = -171$

Substituting the values of x, y, z, and c, into the equation

$x^{14} + y^{14} = c \cdot z^{14}$

$285^{14} + 171^{14} = 6108298594 \cdot 57^{14}$

Sum the left side of the equation

$285^{14} + 171^{14} =$ 2.3343621781973048620100774709 34e+34

Power and product the right side of the equation

$6108298594 \cdot 57^{14} =$ 2.3343621781973048620100774709 34e+34

Give another value of z (z = 312) then

x = r·z = 5x312 = 1560

y = s·z = 3x312 = 936

Substituting the values of x, y, z, and c, into the equation

$x^{14} + y^{14} = c \cdot z^{14}$

$1560^{14} + 936^{14} = 6108298594 \cdot 312^{14}$

Sum the left side of the equation

$1560^{14} + 936^{14} =$ 5.0592317637470821996863661011777e+44

Power and product the right side of the equation

$6108298594 \cdot 312^{14} =$ 5.0592317637470821996863661011777e+44

Solution 1 c = 6108298594

x = 285, y = 171

z = 57

Solution 2

x = 1560, y = 936

z = 312

There is many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^{14} + y^{14} = c \cdot z^{14}$

~~~~~/////~~~~~

8) The Diophantine Equation $x^{23} + y^{23} = c \cdot z^{23}$

*) Find value of x, y, z, which are the whole numbers such as

$x^{23} + y^{23} = c \cdot z^{23}$ (c >2)

~~~~~ solve ~~~~

This equation get four unknowns x, y, z and c.

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

c-value is great than 2

We don't have the list c-value of zeta function $\zeta(s) = r^{23} + s^{23}$. Now give any value of r and s which are positive or negative integers, example

$\zeta(s) = r^{23} + s^{23}$

$\zeta(s) = 4^{23} + 3^{23} = 70462887356491$

$\zeta(s) = 9^{23} + 4^{23} = 886293819002124527359 3$

Or

$\zeta(s) = 17^{23} + (-16)^{23} = 150158087437180027029625688 17$

Now we have three equations with three c-value differents below

$x^{23} + y^{23} = 70462887356491 \cdot z^{23}$ (eq1)

$x^{23} + y^{23} = 886293819002124527359 3 \cdot z^{23}$ (eq2)

$x^{23} + y^{23} = 150158087437180027029625688 17 \cdot z^{23}$ (eq3)

Used the general method for a Generalized Fermat-Wiles Equation (eq1) therefore

$x^{23} + y^{23} = 70462887356491 \cdot z^{23}$ (eq1)

therefore

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 27) then

x = r·z = 4x27 = 108

y = s·z = 3x27 = 81

Substituting the values of x, y, z, into the equation

$x^{23} + y^{23} = 70462887356491 \cdot z^{23}$

$108^{23} + 81^{23} = 70462887356491 \cdot 27^{23}$

Sum the left side of the equation

$108^{23} + 81^{23} =$ 5.8793188128039709282581297296841e+46

Power and product the right side of the equation

$70462887356491 \cdot 27^{23} =$ 5.8793188128039709282581297296841e+46

Used the general method for a Generalized Fermat-Wiles Equation (eq2) therefore

$$x^{23} + y^{23} = 886293819002124527359 3\cdot z^{23} \qquad \text{(eq2)}$$

we have

x = r · z

y = s·z

Give any value of z (z = 13) then

x = r·z = 9x13 = 117

y = s·z = 4x13 = 52

Substituting the values of x, y, z, into the equation

$x^{23} + y^{23} = 886293819002124527359 3\cdot z^{23}$

$117^{23} + 52^{23} = 8862938190021245273593 \cdot 13^{23}$

Sum the left side of the equation

$117^{23} + 52^{23} = 3.7006228287107392044048845713944e+47$

Power and product the right side of the equation

$8862938190021245273593 \cdot 13^{23} =$

$3.7006228287107392044048845713944e+47$

Used the general method for a Generalized Fermat-Wiles Equation (eq3) therefore

$$x^{23} + y^{23} = 150158087437180027029625688 17 \cdot z^{23} \qquad \text{(eq3)}$$

we have

x = r · z

y = s·z

Give any value of z (z = 7) then

x = r·z = 17x7 = 119

y = s·z = -16x7 = -112

Substituting the values of x, y, z, into the equation

$x^{23} + y^{23} = 150158087437180027029625688 17 \cdot z^{23}$

$119^{23} + (-112)^{23} = 15015808743718002702962568817 \cdot 7^{23}$

Sum the left side of the equation

$119^{23} + (-112)^{23} =$ 4.1096387561379585251754870043182e+47

Power and product the right side of the equation

$15015808743718002702962568817 \cdot 7^{23} =$

4.1096387561379585251754870043182e+47

Solution (eq1) c = 70462887356491

x = 108; y = 81; z = 57; z = 27

Solution (eq2) c = 886293819002124527359 3

x = 117; y = 52; z = 13

Solution (eq3) c = 15015808743718002702962568817

x = 117, y = -112; z = 7

Similarly, we have many values of x, y, z, satisfying a Generalized Fermat-Wiles Equation $x^{23} + y^{23} = c \cdot z^{23}$

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CHAPTER 4

The Diophantine Equations $x^k + y^k = z^{k+1}$, (k>=3)

$x^3 + y^3 = z^4$
$x^4 + y^4 = z^5$
$x^5 + y^5 = z^6$
$x^6 + y^6 = z^7$
$x^7 + y^7 = z^8$
$x^8 + y^8 = z^9$
….

There are three unknowns x, y, z. We will spend many hours for these equations by hand calculator or computer, if we have used my general method for the Diophantine equations $x^3 + y^3 = z^4$ … we will find many solutions, also we want to use my general method, we must change the equation $x^3 + y^3 = z^4$
To the form of Fermat-Wiles equation :
$x^3 + y^3 = cz^3$ by c = z
I will explain in chapter Fermat-Wiles equation by general method,

zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

0 $n \to \infty$

~~~~~~/////~~~~~

1) The Diophantine Equation ***$x^3 + y^3 = z^4$***

*) We can change the Diophantine equation above to the form Generalized Fermat-Wiles Equation

$x^n + y^n = c \cdot z^n$

$x^3 + y^3 = z^4$ change to the form $x^3 + y^3 = z \cdot z^3$

get c = z, we have $x^3 + y^3 = c \cdot z^3$

Now we have already used the general method for a Generalized Fermat-Wiles Equation, zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0, this can be expressed by writing
~~~~~~

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

Given any value of r and s (r =11; s = 21) we have

$\zeta(s) = r^3 + s^3 = 11^3 + 21^3 = 10592$

$x^3 + y^3 = 10592 \cdot z^3$

we know that $z = c = 10592$

$x = r \cdot z$

$x = 11x10592 = 116512$

$y = s \cdot z = 21x10592 = 222432$

Substituting the values of x, y, z, and c into the equation

$x^3 + y^3 = c \cdot z^3$

we have $116512^3 + 222432^3 = 10592 \cdot 10592^3$

Sum the left side is equal to

$116512^3 \quad + \quad 222432^3 \quad = \quad 12586700212535296$

And right side is equal to

$10592 \cdot 10592^3 \quad = \quad 12586700212535296$

Given another value of r and s (r =39; s = 47) we have

$\zeta(s) \; = \; r^3 + s^3 \; = \; 39^3 + 47^3 \; = \; 163142$

Replace c = 163142

$x^3 + y^3 = 163142 \cdot z^3$

we know that z = c = 163142

$x \; = \; r \cdot z$

$x \; = \; 39x163142 \; = \; 6362538$

$y \; = \; s \cdot z \; = \; 47x163142 \; = \; 7667674$

Substituting the values of x, y, z, and c into the equation

$x^3 \; + \; y^3 \; = \; c \cdot z^3$

we have $6362538^3 \; + \; 7667674^3 \; = \; 163142 \cdot 163142^3$

Sum the left side is equal to

$6362538^3 \; + \; 7667674^3 \; = \; 70837484158716636289 6$

And product right side is equal to

$163142 \cdot 163142^3 \; = \; 708374841587166362896$

Solution 1

$x = 116512$

$y = 222432$

$z = 10592$

Solution 2

$x = 6362538$

$y = 7667674$

$z = 163142$

With my general method for Generalized Fermat-Wiles Equation by zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0, also we can be found many values of x, y, z, of the Diophantine equation

$$x^3 + y^3 = z^4$$

~~~~~/////~~~~~

2) The Diophantine Equation $x^4 + y^4 = z^5$

*) Find value of x, y, z, which are the whole numbers such as

$$x^4 + y^4 = z^5$$

~~~~ solve ~~~~

First we rewrite this equation $x^4 + y^4 = z^5$ to form Generalized Fermat-Wiles Equation

$x^4 + y^4 = z^5$

$x^4 + y^4 = z \cdot z^4$ put $c = z$ we have

$x^4 + y^4 = c \cdot z^4$

There is a Generalized Fermat-Wiles Equation, we have already used zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0

Second we find the value of zeta function s

$\zeta(s) = r^4 + s^4 = c$

Given any value of r and s (r =17; s = 23) we have

$\zeta(s) = r^4 + s^4 = 17^4 + 23^4 = 363362$

Satisfying c-value into equation

$x^4 + y^4 = 363362 \cdot z^4$

x = r ·z we know that z = c = 363362

x = 17x363362 = 6177154

y = s ·z = 23x363362 = 8357326

Substituting the values of x, y, z, and c into the equation

$x^4 + y^4 = c \cdot z^4$

we have $6177154^4 + 8357326^4 = 363362 \cdot 363362^4$

Sum the left side of the equation is equal to

$6177154^4 + 8357326^4 = 633428407728478238460967683 2$

And product right side is equal to

$363362 \cdot 363362^4 = 6334284077284782384609676832$

Given another value of r and s (r =19; s = 24) we have

$\zeta(s) = r^4 + s^4 = 19^4 + 24^4 = 462097$

Satisfying c-value into equation

$x^4 + y^4 = 462097 \cdot z^4$

$x = r \cdot z$ we know that $z = c = 462097$

$x = 19x462097 = 8779843$

$y = s \cdot z = 24x462097 = 11090328$

Substituting the values of x, y, z, and c into the equation

$x^4 + y^4 = c \cdot z^4$

we have $8779843^4 + 11090328^4 = 462097 \cdot 462097^4$

Sum the left side of the equation is equal to

$8779843^4 + 11090328^4 = 2107005868055224378934645025 7$

And product right side is equal to

$462097 \cdot 462097^4 = 21070058680552243789346450257$

Solution 1

x = 6177154

y = 8357326

z = 363362

Solution 2

x = 8779843

y = 11090328

z = 462097

We can be found many values of x, y, z, of the Diophantine equation by zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0

$x^4 + y^4 = z^5$

~~~~~/////~~~~~

3) The Diophantine Equation $x^5 + y^5 = z^6$

*) Find value of x, y, z, which are the whole numbers such as

$x^5 + y^5 = z^6$

~~~~ solve ~~~~

First we rewrite this equation $x^5 + y^5 = z^6$

to form Generalized Fermat-Wiles Equation

$x^5 + y^5 = z^6$

$x^5 + y^5 = z \cdot z^5$ put $c = z$ we have

$x^5 + y^5 = c \cdot z^5$

There is a Generalized Fermat-Wiles Equation, we have already used zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0 by method

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \rightarrow \infty$$

Second we find the value of zeta function (s)

$\zeta(s) = r^5 + s^5 = c$

Given any value of r and s (r =13; s = 14) we have

$\zeta(s) = r^5 + s^5 = 13^5 + 14^5 = 909117$

Satisfying c-value into equation

$x^5 + y^5 = c \cdot z^5$

$x^5 + y^5 = 909117 \cdot z^5$

we know that $z = c = 909117$

$x = r \cdot z = 13 \cdot 909117 = 11818521$

$y = s \cdot z = 14 \cdot 909117 = 12727638$

Substituting the values of x, y, z, and c into the equation

$x^5 + y^5 = c \cdot z^5$

we have $11818521^5 + 12727638^5 = 909117 \cdot 909117^5$

Sum the left side of the equation is equal to

$11818521^5 + 12727638^5 =$ 5.645711394390725514035898700462 8e+35

And product right side is equal to

$909117 \cdot 909117^5 =$ 5.645711394390725514035898700462 8e+35

Given any value of r and s (r = 18; s = 21) we have

$\zeta(s) = r^5 + s^5 = 18^5 + 21^5 = 5973669$

Satisfying c-value into equation

$x^5 + y^5 = c \cdot z^5$

$x^5 + y^5 = 5973669 \cdot z^5$

we know that $z = c = 5973669$

$x = r \cdot z = 18 \cdot 5973669 = 107526042$

$y = s \cdot z = 21 \cdot 5973669 = 125447049$

Substituting the values of x, y, z, and c into the equation

$x^5 + y^5 = c \cdot z^5$

we have $107526042^5 + 125447049^5 = 5973669 \cdot 5973669^5$

Sum the left side of the equation is equal to

$107526042^5 + 125447049^5 =$ 4.5440900428993601922913375983522e+40

And product right side is equal to

$5973669 \cdot 5973669^5 =$ 4.5440900428993601922913375983522e+40

Solution 1

$x = 11818521$

$y = 12727638$

$z = 909117$

Solution 2

$x = 107526042$

$y \;=\; 125447049$

$z \;=\; 5973669$

We can be found many values of x, y, z, of the Diophantine equation by zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0

$x^5 + y^5 = z^6$

~~~~~/////~~~~~

4) The Diophantine Equation $x^6 + y^6 = z^7$

*) Find value of x, y, z, which are the whole numbers such as

$x^6 + y^6 = z^7$

~~~~ solve ~~~~

First we rewrite this equation $x^6 + y^6 = z^7$

to form Generalized Fermat-Wiles Equation

$x^6 + y^6 = z^7$

$x^6 + y^6 = z \cdot z^6$ put $c = z$ we have

$x^6 + y^6 = c \cdot z^6$

There is a Generalized Fermat-Wiles Equation, we have already used zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0 by method

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

Second we find the value of zeta function (s)

$\zeta(s) = r^6 + s^6 = c$

Given any value of r and s (r =31; s = 42) we have

$\zeta(s) = r^6 + s^6 = 31^6 + 42^6 = 6376535425$

Satisfying c-value into the equation

$x^6 + y^6 = c \cdot z^6$

$x^6 + y^6 = 6376535425 \cdot z^6$

we know that $z = c = 6376535425$

$x = r \cdot z = 31 \cdot 6376535425 = 197672598175$

$y = s \cdot z = 42 \cdot 6376535425 = 267814487850$

Substituting the values of x, y, z, and c into the equation

$x^6 + y^6 = c \cdot z^6$

we have $197672598175^6 + 267814487850^6 = 6376535425 \cdot 6376535425^6$

Sum the left side of the equation is equal to

$197672598175^6 + 267814487850^6 =$

4.2864073143027028021089318985198e+68

And product right side is equal to

$6376535425 \cdot 6376535425^6 =$

4.2864073143027028021089318985198e+68

Given any value of r and s (r = 11; s = 23) we have

$\zeta(s) = r^6 + s^6 = 11^6 + 23^6 = 149807450$

Satisfying c-value into the equation

$x^6 + y^6 = c \cdot z^6$

$x^6 + y^6 = 149807450 \cdot z^6$

we know that z = c = 149807450

$x = r \cdot z = 11 \cdot 149807450 = 1647881950$

$y = s \cdot z = 23 \cdot 149807450 = 3445571350$

Substituting the values of x, y, z, and c into the equation

$x^6 + y^6 = c \cdot z^6$

we have $1647881950^6 + 3445571350^6 = 149807450 \cdot 149807450^6$

Sum the left side of the equation is equal to

$1647881950^6 + 3445571350^6 =$

1.69329989360421988362090264724443e+57

And product right side is equal to

$149807450 \cdot 149807450^6 =$

1.69329989360421988362090264724443e+57

Solution 1

x = 197672598175

y = 267814487850

z = 6376535425

Solution 2

x = 1647881950

y = 3445571350

z = 149807450

We can be found many values of x, y, z, of the Diophantine equation by zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0

~~~~~/////~~~~~

5) The Diophantine Equation $x^7 + y^7 = z^8$

*) Find value of x, y, z, which are the whole numbers such as

$x^7 + y^7 = z^8$

~~~~ solve ~~~~

Similarly above, we rewrite this equation $x^7 + y^7 = z^8$

to form Generalized Fermat-Wiles Equation

$x^7 + y^7 = z^8$

$x^7 + y^7 = z \cdot z^7$ put c = z we have

$x^7 + y^7 = c \cdot z^7$

There is a Generalized Fermat-Wiles Equation, we have already used zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0 by method

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \rightarrow \infty$$

Second we find the value of zeta function (s)

$\zeta(s) = r^7 + s^7 = c$

Given any value of r and s (r =3; s = 4) we have

$\zeta(s) = r^7 + s^7 = 3^7 + 4^7 = 18571$

Satisfying c-value into equation

$x^7 + y^7 = c \cdot z^7$

$x^7 + y^7 = 18571 \cdot z^7$

we know that z = c = 18571

$x = r \cdot z = 3 \cdot 18571 = 55713$

$y = s \cdot z = 4 \cdot 18571 = 74284$

Substituting the values of x, y, z, and c into the equation

$x^7 + y^7 = c \cdot z^7$

we have $55713^7 + 74284^7 = 18571 \cdot 18571^7$

Sum the left side of the equation is equal to

$55713^7 + 74284^7 = 1.4147585263085357173262147916114e+34$

And product right side is equal to

$18571 \cdot 18571^7 = 1.4147585263085357173262147916114e+34$

Given any value of r and s (r = 8; s = 12) we have

$\zeta(s) = r^7 + s^7 = 8^7 + 12^7 = 37928960$

Satisfying c-value into equation

$x^7 + y^7 = c \cdot z^7$

$x^7 + y^7 = 37928960 \cdot z^7$

we know that $z = c = 37928960$

$x = r \cdot z = 8 \cdot 37928960 = 303431680$

$y = s \cdot z = 12 \cdot 37928960 = 455147520$

Substituting the values of x, y, z, and c into the equation

$x^7 + y^7 = c \cdot z^7$

we have $303431680^7 + 455147520^7 = 37928960 \cdot 37928960^7$

Sum the left side of the equation is equal to

$303431680^7 + 455147520^7 =$ 4.2831913541547586528805738062399e+60

And product right side is equal to

$37928960 \cdot 37928960^7 =$ 4.2831913541547586528805738062399e+60

Solution 1

x = 55713

y = 74284

z = 18571

Solution 2

x = 303431680

y = 455147520

z = 37928960

We can be found many values of x, y, z, of the Diophantine equation by zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0

$x^7 + y^7 = z^8$

~~~~~/////~~~~~

6) The Diophantine Equation $x^{12}+ y^{12} = z^{13}$

*) Find value of x, y, z, which are the whole numbers such as

$x^{12} + y^{12} = z^{13}$

~~~~ solve ~~~~

We rewrite this equation $x^{12} + y^{12} = z^{13}$ to form Generalized Fermat-Wiles Equation

$x^{12} + y^{12} = z^{13}$

$x^{12} + y^{12} = z \cdot z^{12}$ put c = z we have

$x^{12} + y^{12} = c \cdot z^{12}$

There is a Generalized Fermat-Wiles Equation, we have already used zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0 by method

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

We find the value of zeta function (s)

$\zeta(s) = r^{12} + s^{12} = c$

Given any value of r and s (r = 6; s = 5) we have

$\zeta(s) = r^{12} + s^{12} = 6^{12} + 5^{12} = 2420922961$

Satisfying c-value into equation

$x^{12} + y^{12} = 2420922961 \cdot z^{12}$

$x = r \cdot z$ we know that $z = c = 2420922961$

$x = 6x\ 2420922961 = 14525537766$

$y = s \cdot z = 5x2420922961 = 12104614805$

Substituting the values of x, y, z, and c into the equation

$x^{12} + y^{12} = c \cdot z^{12}$

we have

$14525537766^{12} + 12104614805^{12} = 2420922961 \cdot 2420922961^{12}$

Sum the left side of the equation is equal to

$14525537766^{12} + 12104614805^{12} =$

9.811888472976315116529605470 38e+121

And product right side is equal to

$2420922961 \cdot 2420922961^{12} =$

9.811888472976315116529605470 38e+121

Given another value of r and s (r =7; s = 3) we have

$\zeta(s) = r^{12} + s^{12} = 7^{12} + 3^{12} = 13841818642$

Satisfying c-value into equation

$x^{12} + y^{12} = 13841818642 \cdot z^{12}$

$x = r \cdot z$ we know that $z = c = 13841818642$

$x = 7x13841818642 = 96892730494$

$y = s \cdot z = 3x13841818642 = 41525455926$

Substituting the values of x, y, z, and c into the equation

$x^{12} + y^{12} = c \cdot z^{12}$

we have

$96892730494^{12} + 41525455926^{12} = 13841818642 \cdot 13841818642^{12}$

Sum the left side of the equation is equal to

$96892730494^{12} + 41525455926^{12} =$

6.8471684394728095660715543159749e+131

And product right side is equal to

$13841818642 \cdot 13841818642^{12} =$

6.8471684394728095660715543159749e+131

Solution 1

x = 14525537766, y = 12104614805

z = 2420922961

Solution 2

x = 96892730494, y = 41525455926

z = 13841818642

We can be found many values of x, y, z, of the Diophantine equation by zeta function ζ(s) is equal to c, and zeta function ζ(c) is equal to 0

$x^{12} + y^{12} = z^{13}$

~~~~~/////~~~~~

Exercises

1*) Find x, y, z, such that the Diophantine equation has form

$x^3 + y^3 = c \cdot z^3$ (c = 602)

2*) Find x, y, z, such that the Diophantine equation has form

$x^4 + y^4 = c \cdot z^4$ (c =79186)

3*) Find x, y, z, such that the Diophantine equation has form

$x^5 + y^5 = c \cdot z^5$ (c = 1436664)

4*) Find x, y, z, such that the Diophantine equation has form

$x^6 + y^6 = c \cdot z^6$ (c =793585)

5*) Find x, y, z, such that the Diophantine equation has form

$x^7 + y^7 = c \cdot z^7$ (c =20310714)

6*) Find x, y, z, such that the Diophantine equation has form

$x^4 + y^4 = c \cdot z^4$ (x =918)

7*) Find x, y, z, such that the Diophantine equation has form
~~~~~

$x^7 + y^7 = c \cdot z^7$ (y = 10714)

8*) Find x, y, z, such that the Diophantine equation has form

$x^7 + y^7 = c \cdot z^7$ (x =2031)

9*) Find x, y, z, such that the Diophantine equation has form

$x^8 + y^8 = c \cdot z^8$

9*) Find x, y, z, such that the Diophantine equation has form

$x^8 + y^8 = c \cdot z^8$ (c = 1745152)

10*) Find x, y, z, such that the Diophantine equation has form

$x^8 + y^8 = c \cdot z^8$ (x = 1173)

11*) Find x, y, z, such that the Diophantine equation has form

$x^8 + y^8 = c \cdot z^8$ (z = 753)

12*) Find x, y, z, such that the Diophantine equation has form

$x^9 + y^9 = c \cdot z^9$

13*) Find x, y, z, such that the Diophantine equation has form

$x^9 + y^9 = c \cdot z^9$ (x = 749)

14*) Find x, y, z, such that the Diophantine equation has form

$x^{10} + y^{10} = c \cdot z^{10}$

15*) Find value of x, y, z, which are the whole numbers such as

$x^{12} + y^{12} = z^{13}$

16*) Find value of x, y, z, which are the whole numbers such as

$x^{14} + y^{14} = z^{15}$

17*) Find value of x, y, z, which are the whole numbers such as

$x^{17} + y^{17} = z^{18}$

18*) Find value of v, x, y, z, which are the whole numbers such as

$v^3 + x^3 + y^3 = z^3$

19*) Find value of v, x, y, z, which are the whole numbers such as

$v^3 + x^3 + y^3 = z^3$

($\zeta(s) = 91$)

20*) Find value of v, x, y, z, which are the whole numbers such as

$v^3 + x^3 + y^3 = z^3$

($\zeta(s) = 189$)

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~~~~~

CHAPTER 5

The Diophantine Equations $x^3 + y^3 + z^3 = k$, (k>=1)

~~~~~/////~~~~~

$$x^3 + y^3 + z^3 = k \quad (k \geq 1)$$

Recently the Mathematicians have proved a new algorithm to find the solutions of the Diophantine equation

$x^3 + y^3 + z^3 = k \ (k \geq 1)$

k is a positive integer and the values of x, y, z, can be any positive integer or negative integer, this equation is very easy prove with k is equal to 3 then $x = y = z = 1$

$$1^3 + 1^3 + 1^3 = 3$$

and another solution is not easy

$$x = y = 4 \text{ and } z = -5$$

$$4^3 + 4^3 + (-5)^3 = k$$

$$64 + 64 - 125 = 3$$

Mathematicians found the general method for this equation when k = 1

We have a multitude the value of x, y, z, by methods|

$$(9t^4)^3+(-9t^4+3t)^3+(-9t^3+1)^3 = 1$$

$$(1-9t^3+648t^4+3888t^9)^3+(-135t^4+3888t^{10})^3+(3t-81t^4-1296t^7-3888t^{10})^3 = 1$$

I found another method, also given us many whole numbers of x, y, z, satisfying the equation :

$x^3 + y^3 + z^3 = k \ (k \geq 2)$, but my method doesn't cover all known of the whole number solutions by method of D.R.
~~~~~

Heath-Brown, W.M. Lioen and H.J.J. Te Riele on a vector computer

My method

$$(2\sqrt[3]{a}+b)^3+(-2\sqrt[3]{a}+b)^3+\left(\sqrt[3]{-12\sqrt[3]{ab^2}}\right)^3 = k$$

Solutions by my method with k = 2

$x^3 + y^3 + z^3 = 2$

$1297^3+(-1295)^3+(-216)^3 = 2$

$279937^3+(-279935)^3+(-7776)^3 = 2$

$60466177^3+(-60466175)^3+(-279936)^3 = 2$

$13060694017^3+(-13060694015)^3+(-10077696)^3 = 2$

$2821109907457^3+(-2821109907457)^3+(-362797056)^3 = 2$

$609359740010497^3+(-609359740010495)^3+(-13060694016)^3 = 2$

….

Solutions by my method with k = 16

$x^3 + y^3 + z^3 = 16$

$20738^3+(-20734)^3+(-1728)^3 = 16$

$35831810^3+(-35831806)^3+(-248832)^3 = 16$

$61917364226^3+(-61917364222)^3+(-35831808)^3 = 16$

$106993205379074^3+(-106993205379070)^3+(-5159780352)^3 = 16$

….

Solutions by my method with k = 54

$x^3 + y^3 + z^3 = 54$

$104979^3+(-104973)^3+(-5832)^3 = 54$

$612220035^3+(-612220029)^3+(-1889568)^3 = 54$

$3570467226627^3+(-3570467226621)^3+(-612220032)^3 = 54$

….

Solutions by my method with $k = 128$

$x^3 + y^3 + z^3 = 128$

$331780^3+(-331772)^3+(-13824)^3 = 128$

$4586471428+(-4586471420)+(-7962624) = 128$

$63403380965380^3+(-63403380965372)^3+(-4586471424)^3 = 128$

…

Solutions by my method with $k = 432$

$x^3 + y^3 + z^3 = 432$

$1679622^3+(-1679610)^3+(-46656)^3 = 432$

$78364164102^3+(-78364164090)^3+(-60466176)^3 = 432$

$3656158440062982^3+(-3656158440062970)^3+(-78364164096)^3 = 432$

…

Solutions by my method with $k = 686$

$x^3 + y^3 + z^3 = 686$

$3111703^3+(-3111689)^3+(-74088)^3 = 686$

$230539333255^3+(-230539333241)^3+(-130691232)^3 = 686$

$17080198121677783^3+(-17080198121677817)^3+(-230539333248)^3=$

686

…

Solutions by my method with $k = 1024$

$x^3 + y^3 + z^3 = 1024$

$5308424^3+(-5308408)^3+(-110592)^3 = 1024$

$587068342280^3+(-587068342264)^3+(-254803968)^3 = 1024$

$64925062108545032^3+(-64925062108554016)^3 + (-587068342272)^3 = 1024$

….

Solutions by my method with k = 1548

$x^3 + y^3 + z^3 = 1548$

$8503065^3+(-8503047^3)+(-157464)^3 = 1548$

$1338925209993^3+(-1338925209975)^3+(-459165024)^3 = 1548$

$2108325192649205 85^3+(-210832519264920567)^3+$
$(-1338925209984)^3 = 1548$

…

Solutions by my method with k = 2662

$x^3 + y^3 + z^3 = 2662$

$18974747^3+(-18974725)^3+(-287496)^3 = 2662$

$5455160701067^3+(-5455160701045)^3+(-1252332576)^3 = 2662$

$1568336880910795787^3+(-1568336880910795765)^3+$
$(-5455160701056)^3 = 2662$

…

There are many whole numbers x, y, z, satisfying the Diophantine equation
$x^3 + y^3 + z^3 = k$ (with k is between 2 to 2662, but my method not cover its)

CHAPTER 6

The Diophantine Equations $v^k + x^k + y^k = z^k$, (k>=2)

1) The Diophantine Equation $v^2 + x^2 + y^2 = z^2$

Recently I have proved a new algorithm to find the solutions of the Diophantine equations: $v^2 + x^2 + y^2 = z^2$

Variables ***v, x, y, z***, can be any positive integer or negative integer, this equation proved by zeta function $\zeta(s)_1$ and zeta function $\zeta(s)_2$ which are equal to c and together zeta function $\zeta(c)_1$ and zeta function $\zeta(c)_2$ are equal to 0, we can be expressed by writing following

Diophantine equations: $v^2 + x^2 + y^2 = z^2$

$\zeta(s)_1 = r_1^2+s_1^2 = c$

$\zeta(s)_2 = r_2^2+s_2^2 = c$ therefore

$\zeta(s)_1 = r_1^2+s_1^2 = 5^2+5^2 = 50$

$\zeta(s)_2 = r_2^2+s_2^2 = 7^2+1^2 = 50$

$\zeta(s)_1 = r_1^2+s_1^2 = 7^2+4^2 = 65$

$\zeta(s)_1 = r_2^2+s_2^2 = 8^2+1^2 = 65$

$\zeta(s)_1 = r_1^2+s_1^2 = 9^2+2^2 = 85$

$\zeta(s)_1 \;=\; r_2^{\,2}+s_2^{\,2} \;=\; 7^2+6^2 \;=\; 85$

~~~~~/////~~~~~

1) The Diophantine Equation $\boldsymbol{v^2 + x^2 + y^2 = z^2}$

1) Find the value of V, X, Y, Z such that

$V^2 + X^2 + Y^2 = Z^2$

~~~~~ solve ~~~~

We can find the values of V, X, Y, Z, by a Generalized Fermat-Wiles Equation
First we find the value of zeta function $\zeta(s)_1$ and zeta function $\zeta(s)_2$ which are equal to c and together zeta function $\zeta(c)_1$ and zeta function $\zeta(c)_2$ are equal to 0,
From the solutions above we have

$\zeta(s)_1 \;=\; r_1^{\,2}+ s_1^{\,2} \;=\; 5^2 + 5^2 \;=\; 50$

$\zeta(s)_2 \;=\; r_2^{\,2}+ s_2^{\,2} \;=\; 7^2 + 1^2 \;=\; 50$

Second we use the new formula for Generalized Fermat-Wiles Equation

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

Third we find the values of x, y, z, of Generalized Fermat-Wiles Equation with $\zeta(s)_1 = \zeta(s)_2 = 50$ We have

$x_1 = r_1 \cdot z = 5 \cdot 742 = 3710$ $(z = 742)$

$y_1 = s_1 \cdot z = 5 \cdot 742 = 3710$

And

$x_2 = r_2 \cdot z = 7 \cdot 742 = 5194$ $(z = 742)$

$y_2 = s_2 \cdot z = 1 \cdot 742 = 742$ therefore

$x_1^2 + y_1^2 = c \cdot z^2$

$3710^2 + 3710^2 = 50 \cdot 724^2 = 27528200$ (1)

And

$x_2^2 + y_2^2 = c \cdot z^2$

$5194^2 \quad + \quad 742^2 \quad = \quad 50 \cdot 724^2 \quad = \quad 27528200$ (2)

From (1) and (2) therefore

$5194^2 \quad + \quad 742^2 \quad = \quad 3710^2 \quad + \quad 3710^2$

Or $5194^2 \quad + \quad 742^2 \quad - \quad 3710^2 \quad = \quad 3710^2 \quad = \quad 13764100$

Comeback the Diophantine equation

$V^2 + X^2 + Y^2 = Z^2$

We have

$V = 5194, \quad X = 742, \quad Y = -3710, \quad Z = 3710$

Given other value of z (z = 3743)

$x_1 = r_1 \cdot z = 5 \cdot 3743 = 18715$

$y_1 = s_1 \cdot z = 5 \cdot 3743 = 18715$

And

$x_2 = r_2 \cdot z = 7 \cdot 3743 = 26201$

$y_2 = s_2 \cdot z = 1 \cdot 3743 = 3743$ therefore

$x_1^2 \quad + \quad y_1^2 \quad = \quad c{\cdot}z^2$

$18715^2 \quad + \quad 18715^2 \quad = \quad 50 \cdot 3743^2 \quad = \quad 700502450$ (1)

And

$x_2^2 \quad + \quad y_2^2 \quad = \quad c{\cdot}z^2$

$26201^2 + 3743^2 = 50 \cdot 3743^2 = 700502450$ (2)

From (1) and (2) therefore

$26201^2 + 3743^2 = 18715^2 + 18715^2$

Or $26201^2 + 3743^2 - 18715^2 = 18715^2$

Comeback the Diophantine equation

Comeback the Diophantine equation

$V^2 + X^2 + Y^2 = Z^2$

We have

$V = 26201, \quad X = 3743, \quad Y = -18715, \quad Z = 18715$

Solution 1

$V = 5194, \quad X = 742, \quad Y = -3710, \quad Z = 3710$

Solution 2

$V = 26201, \quad X = 3743, \quad Y = -18715, \quad Z = 18715$

…

~~~~~/////~~~~~

2) Find the value of V, X, Y, Z such that

$V^2 + X^2 + Y^2 = Z^2$

~~~~~ solve ~~~~

We can find the values of V, X, Y, Z, by a Generalized Fermat-Wiles Equation

First we find the value of zeta function $\zeta(s)_1$ and zeta function $\zeta(s)_2$ which are equal to c and together zeta function $\zeta(c)_1$ and zeta function $\zeta(c)_2$ are equal to 0,

From the solutions above we have

$$\zeta(s)_1 = r_1^2+s_1^2 = 9^2+2^2 = 85$$

$$\zeta(s)_1 = r_2^2+s_2^2 = 7^2+6^2 = 85$$

Second we use the new formula for Generalized Fermat-Wiles Equation

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \to \infty$$

Third we find the values of x, y, z, of Generalized Fermat-Wiles Equation with $\zeta(s)_1 = \zeta(s)_2 = 85$ We have

$$x_1 = r_1 \cdot z = 9 \cdot 259 = 2331 \quad (z = 259)$$

$$y_1 = s_1 \cdot z = 2 \cdot 259 = 518$$

And

$x_2 \; = \; r_2 \cdot \; z \; = \; 7 \cdot \; 259 \quad = \quad 1813 \quad (z \; = \; 259)$

$y_2 \; = \; s_2 \cdot \; z \; = \; 6 \cdot 249 \quad = \quad 1554$ therefore

$x_1^2 \; + \; y_1^2 \; = \; c \cdot z^2$

$2331^2 \; + \; 518^2 \; = \; 85 \cdot 259^2 \; = \; 5701885$ (1)

And

$x_2^2 \; + \; y_2^2 \; = \; c \cdot z^2$

$1813^2 \; + \; 1554^2 \; = \; 85 \cdot 259^2 \; = \; 5701885$ (2)

From (1) and (2) therefore

$5194^2 \; + \; 742^2 \; = \; 3710^2 \; + \; 3710^2$

Or $5194^2 \; + \; 742^2 \; - \; 3710^2 \; = \; 3710^2 \; = \; 13764100$

Comeback the Diophantine equation

$V^2 + X^2 + Y^2 \; = \; Z^2$

We have

$V \; = \; 5194, \quad X \; = \; 742, \quad Y \; = \; -3710, \quad Z \; = \; 3710$

Given other value of z $(z \; = \; 3743)$

$x_1 \; = \; r_1 \cdot \; z \; = \; 5 \; \cdot 3743 \quad = \quad 18715$

$y_1 \; = \; s_1 \cdot \; z \; = \; 5 \; \cdot 3743 \quad = \quad 18715$

And

$x_2 \ = \ r_2 \cdot \ z \ = \ 7 \cdot \ 3743 \ = \ 26201$

$y_2 \ = \ s_2 \cdot \ z \ = \ 1 \ \cdot 3743 \ = \ 3743$ therefore

$x_1^2 \ + \ y_1^2 \ = \ c \cdot z^2$

$18715^2 \ + \ 18715^2 \ = \ 50 \cdot 3743^2 \ = \ 700502450$ (1)

And

$x_2^2 \ + \ y_2^2 \ = \ c \cdot z^2$

$26201^{\ 2} \ + \ 3743^2 \ = \ 50 \cdot 3743^2 \ = \ 700502450$ (2)

From (1) and (2) therefore

$26201^{\ 2} \ + \ 3743^2 \ = \ 18715^2 \ + \ 18715^2$

Or $26201^{\ 2} \ + \ 3743^2 \ - \ 18715^2 \ = \ 18715^2$

Comeback the Diophantine equation

Comeback the Diophantine equation

$V^2 + X^2 + Y^2 \ = \ Z^2$

We have

$V \ = \ 26201, \quad X \ = \ 3743, \quad Y \ = \ -18715, \quad Z \ = \ 18715$

Solution 1

$V \ = \ 5194, \quad X \ = \ 742, \quad Y \ = \ -3710, \quad Z \ = \ 3710$

Solution 2

V = 26201, X = 3743, Y = -18715, Z = 18715

…

~~~~~/////~~~~~

2) The Diophantine Equation $\boldsymbol{v^3 + x^3 + y^3 = z^3}$

Recently I have proved a new algorithm to find the solutions of the Diophantine equations: $\boldsymbol{v^3 + x^3 + y^3 = z^3}$

Variables ***v, x, y, z***, can be any positive integer or negative integer, this equation proved by zeta function $\zeta(s)_1$ and zeta function $\zeta(s)_2$ which are equal to c and together zeta function $\zeta(c)_1$ and zeta function $\zeta(c)_2$ are equal to 0, we can be expressed by writing following

$\zeta(s)_1 = \zeta(s)_2 = c$

Example

$\zeta(s)_1 = r_1^3+s_1^3 = c$

$\zeta(s)_2 = r_2^3+s_2^3 = c$ therefore

$\zeta(s)_1 = \zeta(s)_2 = c$

Example

$\zeta(s)_1 = r_1^3+s_1^3 = 6^3 + (-5)^3 = 91$
~~~~~

$\zeta(s)_2 = r_2^3+s_2^3 = 3^3 + 4^3 = 91$

$\zeta(s)_1 = r_1^3+s_1^3 = 6^3 + (-4)^3 = 152$

$\zeta(s)_2 = r_2^3+s_2^3 = 3^3 + 5^3 = 152$

$\zeta(s)_1 = r_1^3+s_1^3 = 6^3 + (-3)^3 = 189$

$\zeta(s)_2 = r_2^3+s_2^3 = 5^3 + 4^3 = 189$

$\zeta(s)_1 = r_1^3+s_1^3 = 9^3 + (-8)^3 = 217$

$\zeta(s)_2 = r_2^3+s_2^3 = 6^3 + 1^3 = 217$

$\zeta(s)_1 = r_1^3+s_1^3 = 9^3 + (-6)^3 = 513$

$\zeta(s)_2 = r_2^3+s_2^3 = (9/2)^3 + (15/2)^3 = 513$

….

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The Diophantine Equation $\boldsymbol{v^3 + x^3 + y^3 = z^3}$

1) Find the value of V, X, Y, Z, such that

$V^3 + X^3 + Y^3 = Z^3$
~~~~~~

~~~~~ solve ~~~~

We can find the values of V, X, Y, Z, by a Generalized Fermat-Wiles Equation

First we find the value of zeta function $\zeta(s)_1$ and zeta function $\zeta(s)_2$ which are equal to c and together zeta function $\zeta(c)_1$ and zeta function $\zeta(c)_2$ are equal to 0,

From the solutions above we have

$$\zeta(s)_1 = r_1^3+s_1^3 = 6^3 + (-5)^3 = 91$$

$$\zeta(s)_2 = r_2^3+s_2^3 = 3^3 + 4^3 = 91$$

Second we use the new formula for Generalized Fermat-Wiles Equation

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \rightarrow \infty$$

Third we find the values of x, y, z, of Generalized Fermat-Wiles Equation with $\zeta(s)_1 = \zeta(s)_2 = 91$ We have

$$x_1 = r_1 \cdot z = 6 \cdot 17 = 102 \quad (z = 17)$$
~~~~~

$y_1 \; = \; s_1 \cdot z \; = \; (-5) \cdot 17 \; = \; -85 \quad (z = 17)$

And

$x_2 \; = \; r_2 \cdot z \; = \; 3 \cdot 17 \; = \; 51 \quad (z = 17)$

$y_2 \; = \; s_2 \cdot z \; = \; 4 \cdot 17 \; = \; 68 \quad (z = 17)$ therefore

$x_1^3 \; + \; y_1^3 \; = \; c \cdot z^3$

$102^3 + (-85)^3 \; = \; 91 \cdot 17^3 \; = \; 447083 \; (1)$

And

$x_2^3 \; + \; y_2^3 \; = \; c \cdot z^3$

$51^3 \; + \; 68^3 \; = \; 91 \cdot 17^3 \; = \; 447083 \; (2)$

From (1) and (2) therefore

$51^3 \; + \; 68^3 \; = \; 102^3 + (-85)^3$

Or $51^3 \; + \; 68^3 \; + \; 85^3 \; = \; 102^3$

Comeback the Diophantine equation

$V^3 + X^3 + Y^3 \; = \; Z^3$

We have

$V = 51, \quad X = 68, \quad Y = 85, \quad Z = 102$

Given other value of z $(z = 7539)$

$x_1 \; = \; r_1 \cdot z \; = \; 6 \cdot 7539 \; = \; 45234$

$y_1 = s_1 \cdot z = (-5) \cdot 7539 = -37695$

And

$x_2 = r_2 \cdot z = 3 \cdot 7539 = 22617$

$y_2 = s_2 \cdot z = 4 \cdot 7539 = 30156$ therefore

$x_1^3 + y_1^3 = c \cdot z^3$

$45234^3 + (-37695)^3 = 91 \cdot 7539^3 = 38992638395529$ (1)

And

$x_2^3 + y_2^3 = c \cdot z^3$

$22617^3 + 30156^3 = 91 \cdot 7539^3 = 38992638395529$ (2)

From (1) and (2) therefore

$22617^3 + 30156^3 = 45234^3 + (-37695)^3$

Or $22617^3 + 30156^3 + 37695^3 = 45234^3$

Comeback the Diophantine equation

$V^3 + X^3 + Y^3 = Z^3$

We have

$V = 22617, \quad X = 30156, \quad Y = 37695, \quad Z = 45234$

Solution 1

$V = 51, \quad X = 68, \quad Y = 85, \quad Z = 102$

Solution 2

V = 22617, X = 30156, Y = 37695, Z = 45234

…

~~~~~/////~~~~~

The Diophantine Equation $v^3 + x^3 + y^3 = z^3$

2) Find the value of V, X, Y, Z, such that

$V^3 + X^3 + Y^3 = Z^3$

~~~~~ solve ~~~~

We can find the values of V, X, Y, Z, by a Generalized Fermat-Wiles Equation
First we find the value of zeta function $\zeta(s)_1$ and zeta function $\zeta(s)_2$ which are equal to c and together zeta function $\zeta(c)_1$ and zeta function $\zeta(c)_2$ are equal to 0,
From the table above we have

$\zeta(s)_1 = r_1^3+s_1^3 = 6^3 + (-4)^3 = 152$

$\zeta(s)_2 = r_2^3+s_2^3 = 3^3 + 5^3 = 152$

Second we use the new formula for Generalized Fermat-Wiles Equation

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

Third, we find the values of x, y, z, of Generalized Fermat-Wiles Equation with $\zeta(s)_1 = \zeta(s)_2 = 152$ We have

$x_1 = r_1 \cdot z = 6 \cdot 4581 = 27486$ $(z = 4581)$

$y_1 = s_1 \cdot z = (-4) \cdot 4581 = -18324$

And

$x_2 = r_2 \cdot z = 3 \cdot 4581 = 13743$ $(z = 4581)$

$y_2 = s_2 \cdot z = 5 \cdot 4581 = 22905$ therefore

$x_1^3 + y_1^3 = c \cdot z^3$

$27486^3 + (-18324)^3 = 152 \cdot 4581^3 = 14612497951032$ (1)

And

$x_2^3 + y_2^3 = c \cdot z^3$

$13743^3 \;+\; 22905^3 \;=\; 152 \cdot 4581^3 \;=\; 14612497951032$ (2)

From (1) and (2) therefore

$13743^3 \;+\; 22905^3 \;=\; 27486^3 + (-18324)^3$

Or $13743^3 \;+\; 22905^3 \;+\; 18324^3 \;=\; 27486^3$

Comeback the Diophantine equation

$V^3 + X^3 + Y^3 \;=\; Z^3$

We have

$V = 13743, \quad X = 22905, \quad Y = 18324, \quad Z = 27486$

Given other value of z ($z = 93372$)

$x_1 \;=\; r_1 \cdot z \;=\; 6 \cdot 93372 \;=\; 560232$

$y_1 \;=\; s_1 \cdot z \;=\; (-4) \cdot 93372 \;=\; -373488$

And

$x_2 \;=\; r_2 \cdot z \;=\; 3 \cdot 93372 \;=\; 280116$

$y_2 \;=\; s_2 \cdot z \;=\; 5 \cdot 93372 \;=\; 466860$ therefore

$x_1^3 \;+\; y_1^3 \;=\; c \cdot z^3$

$560232^3 + (-373488)^3 \;=\; 152 \cdot 93372^3 \;=\; 123735287581456896$ (1)

And

$x_2^3 + y_2^3 = c \cdot z^3$

$280116^3 + 466860^3 = 152 \cdot 93372^3 = 123735287581456896$ (2)

From (1) and (2) therefore

$280116^3 + 466860^3 = 560232^3 + (-373488)^3$

Or $280116^3 + 466860^3 + 373488^3 = 560232^3$

Comeback the Diophantine equation

$V^3 + X^3 + Y^3 = Z^3$

We have

$280116^3 + 466860^3 + 373488^3 = 560232^3$

Solution 1

V = 13743, X = 22905, Y = 18324, Z = 27486

Solution 2

V = 280116, X = 466860, Y = 373488, Z = 560232

~~~~~/////~~~~~
~~~~~

3) Find the value of V, X, Y, Z such that

$V^3 + X^3 + Y^3 = Z^3$

~~~~~ solve ~~~~

This equation has four unknowns V, X, Y, Z

First step we find zeta function $\zeta(s)_1 = \zeta(s)_2 = c$. Here we see many $\zeta(s)_1 = \zeta(s)_2 = c$ above, in fact there are infinitely the real numbers of the sets of $\zeta(s)_1 = \zeta(s)_2 = c$

Example $\zeta(s)_1 = 9^3 + (-6)^3 = 513$

$\zeta(s)_2 = (9/2)^3 + (15/2)^3 = 513$

Second step find the values of variables which are satisfying with zeta function $\zeta(s)_1 = \zeta(s)_2 = c$

Since $\zeta(s)_1 = 9^3 + (-6)^3 = 513$
~~~~~

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

$x_1 = r_1 \cdot z$ and $y_1 = s_1 \cdot z$

Give any value of z (z = 333332) then

$x_1 = r_1 \cdot z = 9x333332 = 2999988$ and

$y_1 = s_1 \cdot z = -6x333332 = -1999992$

We have $2999988^3 + (-1999992)^3 = 513 \cdot 333332^3$ (1)

Since $\zeta(s)_2 = (9/2)^3 + (15/2)^3 = 513$

$x_2 = r_2 \cdot z$ and $y_2 = s_2 \cdot z$

Give any value of z (z = 333332) then

$x_2 = r_2 \cdot z = (9/2)x333332 = 1499994$ and

$y_2 = s_2 \cdot z = (15/2)x333332 = 2499990$

We have $1499994^3 + 2499990^3 = 513 \cdot 333332^3$ (2)

From (1) and (2) we have

$2999988^3 + (-1999992)^3 = 1499994^3 + 2499990^3$

Replace the terms

$1499994^3 + 2499990^3 + 1999992^3 = 2999988^3$

We verify that

Sum the left terms

$1499994^3 + 2499990^3 + 1999992^3 =$
$2699967600129599827 2$

Power right term

$2999988^3 =$
26999676001295998272

Then

$V^3 + X^3 + Y^3 = Z^3$

$1499994^3 + 2499990^3 + 1999992^3 = 2999988^3$

Solution :

V = 1499994, X = 2499990

Y = 1999992, Z = 2999988

~~~~~/////~~~~~
~~~~~

The Diophantine Equation $v^3 + x^3 + y^3 = z^3$

4) Find the value of V, X, Y, Z such that

$V^3 + X^3 + Y^3 = Z^3$

~~~~~ solve ~~~~

This equation has four unknowns V, X, Y, Z

First step we find zeta function $\zeta(s)_1 = \zeta(s)_2 = c$. Here we see many $\zeta(s)_1 = \zeta(s)_2 = c$ above, in fact there are infinitely the real numbers of the sets of $\zeta(s)_1 = \zeta(s)_2 = c$

Example $\zeta(s)_1 = 9^3 + (-8)^3 = 217$

$\zeta(s)_2 = 6^3 + 1^3 = 217$

Second step find the values of variables which are satisfying with zeta function $\zeta(s)_1 = \zeta(s)_2 = c$

Since $\zeta(s)_1 = 9^3 + (-8)^3 = 217$

We use the new formula for
~~~~~

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

$x_1 = r_1 \cdot z$ and $y_1 = s_1 \cdot z$

Give any value of z (z = 7717171) then

$x_1 = r_1 \cdot z = 9x7717171 = 69454539$ and

$y_1 = s_1 \cdot z = -8x7717171 = -61737368$

We have $69454539^3 + (-61737368)^3 = 217 \cdot 7717171^3$ (1)

Since $\zeta(s)_2 = 6^3 + 1^3 = 217$

$x_2 = r_2 \cdot z$ and $y_2 = s_2 \cdot z$

Give any value of z (z = 7717171) then

$x_2 = r_2 \cdot z = 6x7717171 = 46303026$ and

$y_2 = s_2 \cdot z = 1x7717171 = 7717171$

We have $46303026^3 + 7717171^3 = 217 \cdot 7717171^3$ (2)

From (1) and (2) we have

$69454539^3 + (-61737368)^3 = 46303026^3 + 7717171^3$

Replace the terms

$46303026^3 + 7717171^3 + 61737368^3 = 69454539^3$

We verify that

Sum the left terms

$46303026^3 + 7717171^3 + 61737368^3 =$
335044041827771265192819

Power right term

$69454539^3 =$
335044041827771265192819

Then

$V^3 + X^3 + Y^3 = Z^3$

$46303026^3 + 7717171^3 + 61737368^3 = 69454539^3$

Solution :

V = 46303026, X = 7717171

Y = 61737368, Z = 69454539

~~~~~/////~~~~~
~~~~~

The Diophantine Equation $\boldsymbol{v^3 + x^3 + y^3 = z^3}$

5) Find the value of V, X, Y, Z such that

$V^3 + X^3 + Y^3 = Z^3$

~~~~~ solve ~~~~

We can find the values of V, X, Y, Z, by a Generalized Fermat-Wiles Equation
First we find the value of zeta function $\zeta(s)_1$ and zeta function $\zeta(s)_2$ which are equal to c and together zeta function $\zeta(c)_1$ and zeta function $\zeta(c)_2$ are equal to 0,
From the table above we have

$\zeta(s)_1 = r_1^3+s_1^3 = 6^3 + (-3)^3 = 189$

$\zeta(s)_2 = r_2^3+s_2^3 = 5^3 + 4^3 = 189$

Second we use the new formula for Generalized Fermat-Wiles Equation
~~~~~

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

Third we find the values of x, y, z, of Generalized Fermat-Wiles Equation with $\zeta(s)_1 = \zeta(s)_2 = 189$ We have

$x_1 = r_1 \cdot z = 6 \cdot 8763 = 52578$ ($z = 8763$)

$y_1 = s_1 \cdot z = (-3) \cdot 8763 = -26289$

And

$x_2 = r_2 \cdot z = 4 \cdot 8763 = 35052$ ($z = 8763$)

$y_2 = s_2 \cdot z = 5 \cdot 8763 = 43815$ therefore

$x_1^3 + y_1^3 = c \cdot z^3$

$52578^3 + (-26289)^3 = 189 \cdot 8763^3 = 127180415428983$ (1)

And

$x_2^3 + y_2^3 = c \cdot z^3$

$35052^3 \ + \ 43815^3 \ = \ 189 \cdot 8763^3 \ = \ 127180415428983$ (2)

From (1) and (2) therefore

$35052^3 \ + \ 43815^3 \ = \ 52578^3 + (-26289)^3$

Or $35052^3 \ + \ 43815^3 \ + \ 26289^3 \ = \ 52578^3$

Comeback the Diophantine equation

$V^3 + X^3 + Y^3 \ = \ Z^3$

$35052^3 \ + \ 43815^3 \ + \ 26289^3 \ = \ 52578^3$

Solution

V = 35052, X = 43815,

Y = 26289, Z = 52578

~~~~~/////~~~~~

The Diophantine Equation $\boldsymbol{v^3 + x^3 + y^3 \ = \ z^3}$

6) Find the value of v, x, y, such that

$v^3 + x^3 + y^3 \ = \ 88434^3$

~~~~~ solve ~~~~

This equation $v^3 + x^3 + y^3 = 88434^3$

Rewrite $v^3 + x^3 = 88434^3 - y^3$

Returning to the form Generalized Fermat-Wiles equation

$x^n + y^n = c \cdot z^n$

Then

$$v^3 + x^3 = c \cdot z^3 \quad (1)$$

$$88434^3 - y^3 = c \cdot z^3 \quad (2)$$

Similarly above, we find the value of zeta function $\zeta(s)_1$ and zeta function $\zeta(s)_2$ which are equal to c and together zeta function $\zeta(c)_1$ and zeta function $\zeta(c)_2$ are equal to 0, and we use the method for Generalized Fermat-Wiles Equation

$$\zeta(s)_1 = r_1^3+s_1^3 = 4^3 + 5^3 = 189$$

$$\zeta(s)_2 = r_2^3+s_2^3 = 6^3 + (-3)^3 = 189$$

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

$\zeta(s)_1 = \zeta(s)_2 = c = 189$ above

We have $x = rz \rightarrow z = x/r$

$z = 88434/6 = 14739$

$y = s_2 \cdot z = 14739 \cdot (-3) = -44217$

$x = r_1 \cdot z = 14739 \cdot 4 = 58956$

$v = s_1 \cdot z = 14739 \cdot 5 = 73695$

By substituting the equation $v^3 + x^3 + y^3 = 88434^3$

We verify that

$73695^3 + 58956^3 + 44217^3 = 88434^3$

Sum the left terms

$73695^3 + 58956^3 + 44217^3 = 691604495730504$

Power right term we have

$88434^3 = 691604495730504$

Use another $\zeta(s)_1 = \zeta(s)_2 = c$ above

$\zeta(s)_1 = r_1^3+s_1^3 = 9^3 + (-8)^3 = 217$

$\zeta(s)_2 = r_2^3+s_2^3 = 6^3 + 1^3 = 217$

Similarly we have

$v = 78608$

$x = 58956$

$y = 9826$

By substituting the equation $v^3 + x^3 + y^3 = 88434^3$

We verify that

$78608^3+58956^3+9826^3 = 88434^3$

Sum the left terms

$78608^3+58956^3+9826^3 = 691604495730504$

Power right term we have

$88434^3 = 691604495730504$

Solution 1

v = 73695

x = 58956

y = 44217

solution 2

v = 78608

x = 58956

y = 9826

~~~~~/////~~~~~

The Diophantine Equation $\boldsymbol{v^3 + x^3 + y^3 = z^3}$

7) Find the value of v, x, y, such that

$v^3 + x^3 + y^3 = 1581336^3$

~~~~~ solve ~~~~

This equation $v^3 + x^3 + y^3 = 1581336^3$

Rewrite $v^3 + x^3 = 1581336^3 - y^3$

Returning to the form Generalized Fermat-Wiles equation

$x^n + y^n = c \cdot z^n$

Then

$v^3 + x^3 = c \cdot z^3$ (1)

$1581336^3 - y^3 = c \cdot z^3$ (2)

Similarly above, we find the value of zeta function $\zeta(s)_1$ and zeta function $\zeta(s)_2$ which are equal to c and together zeta function $\zeta(c)_1$ and zeta function $\zeta(c)_2$ are equal to 0, and we use the method for Generalized Fermat-Wiles Equation below

$\zeta(s)_1 = r_1^3+s_1^3 = 6^3 + (-3)^3 = 189$

$\zeta(s)_2 = r_2^3+s_2^3 = 5^3 + 4^3 = 189$

$\zeta(s)_1 = r_1^3+s_1^3 = 9^3 + (-8)^3 = 217$

$\zeta(s)_2 = r_2^3+s_2^3 = 6^3 + 1^3 = 217$

$\zeta(s)_1 = r_1^3+s_1^3 = 9^3 + (-6)^3 = 513$

$\zeta(s)_2 = r_2^3+s_2^3 = (9/2)^3 + (15/2)^3 = 513$

….

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

** $\zeta(s)_1 = \zeta(s)_2 = c = 189$ above

$\zeta(s)_1 = r_1^3+s_1^3 = 6^3 + (-3)^3 = 189$

$\zeta(s)_2 = r_2^3+s_2^3 = 5^3 + 4^3 = 189$

We have $x = rz \rightarrow z = x/r$

$z = 1581336/6 = 263556$

$y = s_2 \cdot z = 263556 \cdot (-3) = -790668$

$x = r_1 \cdot z = 263556 \cdot 4 = 1054224$

$v = s_1 \cdot z = 263556 \cdot 5 = 1317780$

By substituting the equation $v^3 + x^3 + y^3 = 1581336^3$

We verify that

$1317780^3 + 1054224^3 + 790668^3 = 1581336^3$

Sum the left terms

$1317780^3 + 1054224^3 + 790668^3 = 3954326033991661056$

Power right term we have

$1581336^3 = 3954326033991661056$

We have

$v = 1317780, \quad x = 1054224, \quad y = 790668$

** Using another value of $\zeta(s)_1 = \zeta(s)_2 = c$ above

$\zeta(s)_1 = r_1^3 + s_1^3 = 9^3 + (-8)^3 = 217$

$\zeta(s)_2 = r_2^3 + s_2^3 = 6^3 + 1^3 = 217$

Similarly we have

We have $x = rz \rightarrow z = x/r$

$z = 1581336/9 = 175704$

$y = s_2 \cdot z = 175704 \cdot (-8) = -1405632$

$x = r_1 \cdot z = 175704 \cdot 6 = 1054224$

$v = s_1 \cdot z = 175704 \cdot 1 = 175704$

By substituting the equation $v^3 + x^3 + y^3 = 1581336^3$

We verify that

$175704^3 + 1054224^3 + 1405632^3 = 1581336^3$

Sum the left terms

$175704^3 + 1054224^3 + 1405632^3 = 3954326033991661056$

Power right term we have

$1581336^3 = 3954326033991661056$

Solution 1

v = 1317780, x = 1054224, y = 790668

Solution 2

v = 175704, x = 1054224, y = 1405632

~~~~~/////~~~~~

Exercises

1*) Find v, x, y, z, such that the Diophantine equation has form

$v^3 + x^3 + y^3 = z^3$

2*) Find v, x, y, z, such that the Diophantine equation has form

$v^3 + x^3 + y^3 = z^3$ give v = 279

3*) Find v, x, y, z, such that the Diophantine equation has form
~~~~~

$v^3 + x^3 + y^3 = z^3$ Give $x = 3321$

4*) Find v, x, y, z, such that the Diophantine equation has form

$v^3 + x^3 + y^3 = z^3$ Give $y = 23472$

5*) Find v, x, y, z, such that the Diophantine equation has form

$v^3 + x^3 + y^3 = z^3$

Give $z = 677934$

6*) Find v, x, y, z, such that the Diophantine equation has form

$$x^3 + y^3 + z^3 = k$$

Give $k = 3456$

7*) Find v, x, y, z, such that the Diophantine equation has form

$$x^3 + y^3 + z^3 = k$$

Give $k = 4394$

8*) Find v, x, y, z, such that the Diophantine equation has form

$$x^3 + y^3 + z^3 = k$$

Give $k = 5488$

CHAPTER 7

The Diophantine Equations
$v^n + x^n + y^n = d \cdot z^n$

1) The Diophantine Equation $v^n + x^n + y^n = d \cdot z^n$

I have proved a new formula in first books Fermat's last Theorem , We use this formula to find the solutions of the Diophantine equation

$v^n + x^n + y^n = d \cdot z^n$

Variables ***v, x, y, z***, and ***d*** are the whole numbers, this equation proved by

zeta function ζ(s) is equal to c and zeta function ζ(c) is equal to 0, and

zeta function ζ(s) is equal to c and zeta function ζ(c) is not equal to 0, we can be expressed by writing following

$\zeta(s)_1 = r_1^n + s_1^n = c$ therefore

$s_1 = \sqrt[n]{c - r_1^n}$ is rational number

$\zeta(s)_2 = r_2^n + s_2^n = c$ therefore

$s_2 = \sqrt[n]{c - r_2^n}$ is irrational number

And $c_1 = c_2 = c$, then we have $\zeta(s)_1 = \zeta(s)_2$

$x_1 = r_1 \cdot z$

$y_1 = s_1 \cdot z$ (1)

And

$x_2 = r_2 \cdot z$

$y_2 = s_2 \cdot z = \sqrt[n]{c - r_2^n} \cdot z$ (2)

By (1) and (2)

$x_1^n + y_1^n = x_2^n + y_2^n = c \cdot z^n$

$x_1^n + y_1^n = x_2^n + (c - r^n) \cdot z^n$

let $d = (c - r^n)$

$x_1^n + y_1^n = x_2^n + d \cdot z^n$

Replace this terms we have

$\boldsymbol{v^n + x^n + y^n = d \cdot z^n}$

~~~~~/////~~~~~

The Diophantine Equation $\boldsymbol{v^n + x^n + y^n = d \cdot z^n}$

1) Find the value of V, X, Y, Z and d which are whole numbers such that

$V^3 + X^3 + Y^3 = d \cdot Z^3$
~~~~~

We rewrite This Diophantine equation $V^3 + X^3 + Y^3 = d \cdot Z^3$ to

$V^3 + X^3 = -Y^3 + d \cdot Z^3$

Using method above

$\zeta(s)_1 = r_1^n + s_1^n = c$ therefore

$s_1 = \sqrt[n]{c - r_1^n}$ is rational number

$\zeta(s)_2 = r_2^n + s_2^n = c$ therefore

$s_2 = \sqrt[n]{c - r_2^n}$ is irrational number

And $c_1 = c_2 = c$, then we have $\zeta(s)_1 = \zeta(s)_2$

$x_1 = r_1 \cdot z$

$y_1 = s_1 \cdot z$

And

$x_2 = r_2 \cdot z$

$y_2 = s_2 \cdot z = \sqrt[n]{c - r_2^n} \cdot z$

We choose any value of zeta function s $\zeta(s)_1$ and any value of z
Example: $\zeta(s)_1 = r_1^3 + s_1^3 = 9^3 + 4^3 = 793$ and $z = 991$
We have

$x_1 = r_1 \cdot z = 9 \cdot 991 = 8919$

$y_1 = s_1 \cdot z = 4 \cdot 991 = 3964$

Next we choose any value of r_2, $\zeta(s)_2 = \zeta(s)_1 = 793$

Example : $r_2 = 8$
We have

$x_2 = r_2 \cdot z = 8 \cdot 991 = 7928$ ($z = 991$ above)

$$y_2 = s_2 \cdot z = \sqrt[3]{c - r_2^3} \cdot z = \sqrt[3]{281} \cdot 991 \qquad (2)$$

Return to Generalized Fermat-Wiles Equation, we have

$$x_1^3 + y_1^3 = x_2^3 + y_2^3 = c \cdot z^3$$

$$8919^3 + 3964^3 = 7928^3 + (\sqrt[3]{281} \cdot 991)^3 = 793 \cdot 991^3$$

$$= 771781120903$$

Replace this terms we have

Diophantine equation $V^3 + X^3 + Y^3 = d \cdot Z^3$

$$8919^3 + 3964^3 - 7928^3 = 281 \cdot 991^3$$

Checking:

$8919^3 + 3964^3 - 7928^3 = 273481078151$
And right
$281 \cdot 991^3 = 273481078151$

Solution

V = 8919, X = 3964
Y = -7928, Z = 991, d = 281

The Diophantine Equation $v^n + x^n + y^n = \mathbf{d} \cdot z^n$

2) Find the value of V, X, Y, Z and d which are whole numbers such that

$V^3 + X^3 + Y^3 = d \cdot Z^3$

We rewrite this Diophantine equation $V^3 + X^3 + Y^3 = d \cdot Z^3$ to

$V^3 + X^3 = -Y^3 + d \cdot Z^3$

Using method above

$\zeta(s)_1 = r_1^n + s_1^n = c$ therefore

$s_1 = \sqrt[n]{c - r_1^n}$ is rational number

$\zeta(s)_2 = r_2^n + s_2^n = c$ therefore

$s_2 = \sqrt[n]{c - r_2^n}$ is irrational number

And $c_1 = c_2 = c$, then we have $\zeta(s)_1 = \zeta(s)_2$

$x_1 = r_1 \cdot z$

$y_1 = s_1 \cdot z$

And

$x_2 = r_2 \cdot z$

$y_2 = s_2 \cdot z = \sqrt[n]{c - r_2^n} \cdot z$

We choose any value of zeta function s $\zeta(s)_1$ and any value of z
Example: $\zeta(s)_1 = r_1^3 + s_1^3 = -11^3 + 12^3 = 397$ and $z = 4426$
We have

$x_1 = r_1 \cdot z = -11 \cdot 4426 = -48686$

$y_1 = s_1 \cdot z = 12 \cdot 4426 = 53112$

Next we choose any value of r_2, $\zeta(s)_2 = \zeta(s)_1 = 397$

Example : $r_2 = 7$
We have

$x_2 = r_2 \cdot z = 7 \cdot 4426 = 30982$ ($z = 4426$ above)

$y_2 = s_2 \cdot z = \sqrt[3]{c - r_2^3} \cdot z = \sqrt[3]{54} \cdot 4426$
(2)

Return to Generalized Fermat-Wiles Equation, we have

$x_1^3 + y_1^3 = x_2^3 + y_2^3 = c \cdot z^3 - 48686^3 + 53112^3 = 30982^3 + (\sqrt[3]{54} \cdot 4426)^3 = 397 \cdot 4426^3 = 34421099248072$

Replace this terms we have

Diophantine equation $V^3 + X^3 + Y^3 = d \cdot Z^3$

$-48686^3 + 53112^3 - 30982^3 = 54 \cdot 4426^3$

Checking:

$-48686^3 + 53112^3 - 30982^3 = 4681963121904$

And right

$54 \cdot 4426^3 = 4681963121904$

Solution

$V = -48686, \quad X = 53112$

$Y = -30982, \quad Z = 4426, \quad d = 54$

~~~~~/////~~~~~

The Diophantine Equation $v^n + x^n + y^n = d \cdot z^n$

3) Find the value of V, X, Y, Z and d which are whole numbers such that

$V^4 + X^4 + Y^4 = d \cdot Z^4$
~~~~~

We rewrite this Diophantine equation $V^4 + X^4 + Y^4 = d \cdot Z^4$ to

$V^4 + X^4 = -Y^4 + d \cdot Z^4$

Using method above we have

$\zeta(s)_1 = r_1^n + s_1^n = c$ therefore

$s_1 = \sqrt[n]{c - r_1^n}$ is rational number

$\zeta(s)_2 = r_2^n + s_2^n = c$ therefore

$s_2 = \sqrt[n]{c - r_2^n}$ is irrational number

And $c_1 = c_2 = c$, then we have $\zeta(s)_1 = \zeta(s)_2$

$x_1 = r_1 \cdot z$

$y_1 = s_1 \cdot z$

And

$x_2 = r_2 \cdot z$

$y_2 = s_2 \cdot z = \sqrt[n]{c - r_2^n} \cdot z$

We choose any value of zeta function s $\zeta(s)_1$ and any value of z
Example: $\zeta(s)_1 = r_1^4 + s_1^4 = 7^4 + 3^4 = 2482$ and $z = 7531$
We have

$$x_1 = r_1 \cdot z = 7 \cdot 7531 = 52717$$

$$y_1 = s_1 \cdot z = 3 \cdot 7531 = 22593$$

Next we choose any value of r_2, $\zeta(s)_2 = \zeta(s)_1 = 2482$

Example : $r_2 = -6$
We have

$x_2 = r_2 \cdot z = -6 \cdot 7531 = -45186$ (z = 7531 above)

$y_2 = s_2 \cdot z = \sqrt[4]{c - r_2^4} \cdot z = \sqrt[4]{3778} \cdot 7531$
(2)

Return to Generalized Fermat-Wiles Equation, we have

$x_1^4 + y_1^4 = x_2^4 + y_2^4 = c \cdot z^4$

$52717^4 + 22593^4 = -45186^4 + (\sqrt[4]{3778} \cdot 7531)^4 = 2482 \cdot 7531^4$

$= 7983849976205039122$

Replace this terms we have

Diophantine equation $V^4 + X^4 + Y^4 = d \cdot Z^4$

$52717^4 + 22593^4 + 45186^4 = 3778 \cdot 7531^4$

Checking:

$52717^4 + 22593^4 + 45186^4 = 1215269347707600233 8$
And right

$$3778 \cdot 7531^4 = 1215269347707600233 8$$

Solution

$V = 52717, \quad X = 22593$
$Y = 45186, \quad Z = 7531, \quad d = 3778$

The Diophantine Equation $v^n + x^n + y^n = d \cdot z^n$

4) Find the value of V, X, Y, Z and d which are whole numbers such that

$V^5 + X^5 + Y^5 = d \cdot Z^5$

We rewrite this Diophantine equation $V^5 + X^5 + Y^5 = d \cdot Z^5$ to

$V^5 + X^5 = -Y^5 + d \cdot Z^5$

Using method above we have

$\zeta(s)_1 = r_1^n + s_1^n = c$ therefore

$s_1 = \sqrt[n]{c - r_1^n}$ is rational number

$\zeta(s)_2 = r_2^n + s_2^n = c$ therefore

$s_2 = \sqrt[n]{c - r_2^n}$ is irrational number

And $c_1 = c_2 = c$, then we have $\zeta(s)_1 = \zeta(s)_2$

$x_1 = r_1 \cdot z$

$y_1 = s_1 \cdot z$

And

$x_2 = r_2 \cdot z$

$y_2 = s_2 \cdot z = \sqrt[n]{c - r_2^n} \cdot z$

We choose any value of zeta function s $\zeta(s)_1$ and any value of z

Example: $\zeta(s)_1 = r_1^5 + s_1^5 = 5^5 + 8^5 = 35893$ and $z = 882$
We have

$x_1 = r_1 \cdot z = 5 \cdot 882 = 4410$

$y_1 = s_1 \cdot z = 8 \cdot 882 = 7056$

Next we choose any value of r_2, $\zeta(s)_2 = \zeta(s)_1 = 35893$

Example : $r_2 = -9$
We have

$x_2 = r_2 \cdot z = -9 \cdot 882 = -7938$ ($z = 882$)

$y_2 = s_2 \cdot z = \sqrt[5]{c - r_2^5} \cdot z = \sqrt[5]{94942} \cdot 882$ (2)

Return to Generalized Fermat-Wiles Equation, we have

$x_1^5 + y_1^5 = x_2^5 + y_2^5 = c \cdot z^5$

$4410^5 + 7056^5 = -7938^5 + (\sqrt[5]{94942} \cdot 882)^5 = 35893 \cdot 882^5 = 19158110974418191776$

Replace this terms we have

Diophantine equation $V^5 + X^5 + Y^5 = d \cdot Z^5$

$4410^5 + 7056^5 + 7938^5 = 94942 \cdot 882^5$

Checking:

$4410^5 + 7056^5 + 7938^5 = 50675880314635498944$
And right
$94942 \cdot 882^5 = 50675880314635498944$
Solution

$V = 4410$, $X = 7056$
$Y = 7938$, $Z = 882$, $d = 94942$

CHAPTER 8

The Diophantine Equations With *ζ(s)* is not a Whole Number

Sum of two fractional numbers and ζ(s) = c is the fractional number, too

Example

$(1/3)^3 + (2/3)^3 = 1/3$

$(4/3)^3 + (7/3)^3 = 407/27$

$(2/5)^3 + (3/5)^3 = 35/125 = 7/25$

$(7/5)^3 + (8/5)^3 = 855/125 = 171/25$

$(11/4)^3 + (9/4)^3 = 2060/64 = 515/4$

$(2/7)^3 + (3/7)^3 = 35/343 = 5/49$

$(1/9)^3 + (2/9)^3 = 9/729 = 1/81$

$(2/9)^3 + (7/9)^3 = 351/729 = 13/27$

$(4/9)^3 + (5/9)^3 = 189/729 = 7/27$

$(8/9)^3 + (7/9)^3 = 855/729 = 95/81$

$(13/9)^3 + (14/9)^3 = 4941/729 = 61/9$

(15/12) + (18/12) = 9207/1728 = 341/64 ….

We have $x^n + y^n = (c/a)\, z^n$

In chapter we use zeta function $\zeta(s) = c$ and zeta function $\zeta(c) = 0$, but c is not whole numbers. Also it is very important for solutions of the Diophantine Equations have forms

$$ax^n + by^n = cz^n$$

$$(n \geq 2) \text{ and } a = b$$

* First step we find value of x, y, z, by zeta function $\zeta(s) = c$ and $\zeta(c) = 0$ but c is not whole numbers

*) Second step we multiply both sides of the equation by the denominator of c for c is the whole number

$$x^n + y^n = (c/a)\, z^n \quad = \quad ax^n + ay^n = c\, z^n$$

*) Third step we have the equation $ax^n + by^n = cz^n$ or invert we divide both sides by a or b (a = b)

~~~~~/////~~~~~

zeta function ζ(s) is equal to c and c is not whole numbers

1) Find value x, y, z

$$3x^3 + 3y^3 = z^3$$

~~~~~ Solve ~~~~~

First we divide both sides by 3

$$x^3 + y^3 = (1/3)\, z^3$$

Next we find $\zeta(s) = r^3 + s^3 = c$ and $\zeta(c) = 0$

Here c is equal to 1/3, but 3 is not 3th powers of any number, then we multiply 3 by any number (example 9) for product equal 3th powers

c = (1/3)x(9/9) = 9/27

we find $\zeta(s) = r^3 + s^3 = 9/27$

$\zeta(s) = (1/3)^3 + (2/3)^3 = 9/27$ (c is not whole)

Already use the general method

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \rightarrow \infty$$

$\zeta(s) = (1/3)^3 + (2/3)^3 = 9/27$

x = r ·z

y = s·z

Give any value of z (z = 21) which is common factor by 3

$x = r \cdot z = (1/3)x21 = 7$

$y = s \cdot z = (2/3)x21 = 14$

We verify that

$3x^3 + 3y^3 = z^3$

$3x7^3 + 3x14^3 = 21^3 = 9261$

Give another value of z (z = 528) which is common factor by 3

$x = r \cdot z = (1/3)x528 = 176$

$y = s \cdot z = (2/3)x528 = 352$

We verify that

$3x^3 + 3y^3 = z^3$

$3x176^3 + 3x352^3 = 528^3 = 147197952$

Similarly we have many solutions of this equation

Solution 1	Solution 2
$x = 7$	$x = 176$
$y = 14$	$y = 352$
$z = 21$	$z = 528$

~~~~~/////~~~~~
~~~~~

Zeta function ζ(s) is equal to c and c is not whole numbers

2) Find value x, y, z

$25x^3 + 25y^3 = 7z^3$

~~~~~ Solve ~~~~~

First we divide both sides of equation $25x^3 + 25y^3 = 7z^3$ by 25

$x^3 + y^3 = (7/25) z^3$

Next we find $\zeta(s) = r^3 + s^3 = c$ and $\zeta(c) = 0$

Here c is equal to 7/25, but 25 is not 3th powers of any number, then we multiply 25 by any number (example 5) for product equal 3th powers

$c = (7/25)x(5/5) = 35/125$

we find $\zeta(s) = r^3 + s^3 = 35/125$

$\zeta(s) = (2/5)^3 + (3/5)^3 = 35/125$ (c is not whole)

Already use the general method
~~~~~

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

$\zeta(s) = (2/5)^3 + (3/5)^3 = 35/125$

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 45) which is common factor by 5

$x = r \cdot z = (2/5)x45 = 18$

$y = s \cdot z = (3/5)x45 = 27$

We verify that

$25x^3 + 25y^3 = 7z^3$

$25x18^3 + 25x27^3 = 7x45^3 = 637875$

Give another value of z (z = 375) which is common factor by 5

$x = r \cdot z = (2/5)x375 = 150$

$y = s \cdot z = (3/5)x375 = 225$

We verify that

$25x^3 + 25y^3 = 7z^3$

$25x150^3 + 25x225^3 = 7x375^3 = 369140625$

Similarly we have many solutions of this equation

Solution 1	Solution 2
$x = 18$	$x = 150$
$y = 27$	$y = 225$
$z = 45$	$z = 375$

~~~~~/////~~~~~

zeta function ζ(s) is equal to c and c is not whole numbers

3) Find value x, y, z

$49x^3 + 49y^3 = 5z^3$

~~~~~ Solve ~~~~~

First we divide both sides of equation $49x^3 + 49y^3 = 5z^3$ by 49

$x^3 + y^3 = (5/49) z^3$

Next we find $\zeta(s) = r^3 + s^3 = c$ and $\zeta(c) = 0$

Here c is equal to 5/49, but 49 is not 3^{th} powers of any number, then we multiply 49 by any number (example 7) for product equal 3^{th} powers

$$c = (5/49)x(7/7) = 35/343$$

we find $\zeta(s) = r^3 + s^3 = 35/343$

$$\zeta(s) = (2/7)^3 + (3/7)^3 = 35/343 \text{ (c is not whole)}$$

Already use the general method

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

$$\zeta(s) = (2/7)^3 + (3/7)^3 = 35/343$$

$$x = r \cdot z$$

$$y = s \cdot z$$

Give any value of z (z = 42) which is common factor by 7

$$x = r \cdot z = (2/7)x42 = 12$$

$y = s \cdot z = (3/7) \times 42 = 18$

We verify that

$$49x^3 + 49y^3 = 5z^3$$

$$49 \times 12^3 + 49 \times 18^3 = 5 \times 42^3 = 370440$$

Give another value of z (z = 91) which is common factor by 7

$x = r \cdot z = (2/7) \times 91 = 26$

$y = s \cdot z = (3/7) \times 91 = 39$

We verify that

$$49x^3 + 49y^3 = 5z^3$$

$$49 \times 26^3 + 49 \times 39^3 = 5 \times 91^3 = 3767855$$

Similarly we have many solutions of this equation

Solution 1	Solution 2
$x = 12$	$x = 26$
$y = 18$	$y = 39$
$z = 42$	$z = 91$

~~~~~/////~~~~~
~~~~~

zeta function ζ(s) is equal to c and c is not whole numbers

4) Find value a, b, c, x, y, z

$ax^3 + by^3 = cz^3$

Given a = b

~~~~~ Solve ~~~~~

First we divide both sides of equation $ax^3 + by^3 = cz^3$ by a

$x^3 + y^3 = (c/a) z^3$

Next we find $\zeta(s) = r^3 + s^3 = c$ and $\zeta(c) = 0$

Here c is equal to (c/a), get any faction of r and s we have

$\zeta(s) = (13/9)^3 + (14/9)^3 = 4941/729 = 61/9$

$\zeta(s) = 4941/729 = 61/9$ substituting $\zeta(s)$ value into the equation

$x^3 + y^3 = cz^3$

$x^3 + y^3 = (61/9) \cdot z^3$

We use the general method
~~~~~

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

$$\zeta(s) = (13/9)^3 + (14/9)^3 = 4941/729 = 61/9$$

$$x = r \cdot z$$

$$y = s \cdot z$$

Give any value of z (z = 63) which is common factor by 9

$$x = r \cdot z = (13/9)x63 = 91$$

$$y = s \cdot z = (14/9)x63 = 98$$

c-value is not whole number then we can rewrite this equation

$$x^3 + y^3 = (61/9) \cdot z^3$$

$$9x^3 + 9y^3 = 61 \cdot z^3$$

We verify that

$$9x^3 + 9y^3 = 61z^3$$

$9x91^3 + 9x98^3 = 61x63^3$

Sum left terms

$9x91^3 + 9x98^3 = 15252867$

Product right terms

$61x63^3 = 15252867$

Give another value of z (z = 144) which is common factor by 9

$x = r \cdot z = (13/9)x144 = 208$

$y = s \cdot z = (14/9)x144 = 224$

We verify that

$9x^3 + 9y^3 = 61z^3$

$9x208^3 + 9x224^3 = 61x144^3$

Sum left terms

$9x208^3 + 9x224^3 = 182145024$

Product right terms

$61x144^3 = 182145024$

Similarly we have many solutions of this equation

Solution 1 Solution 2

$a = b = 9, \ c = 61$

$x = 91$ $x = 208$

$y = 98$ $y = 224$

$z = 63$ $z = 144$

~~~~~/////~~~~~

zeta function ζ(s) is equal to c and c is not whole numbers

5) Find value a, b, c, x, y, z

$ax^4 + by^4 = cz^4$

Given $a = b$

~~~~~ Solve ~~~~~

First we divide both sides of equation $ax^4 + by^4 = cz^4$ by a

$x^4 + y^4 = (c/a)\, z^4$

Next we find $\zeta(s) = r^4 + s^4 = c$ and $\zeta(c) = 0$

Here c is equal to (c/a), get any faction of r and s we have

$\zeta(s) = (27/21)^4 + (18/21)^4 = 636417/194481$

$= 7857/2401$

$\zeta(s) = 7857/2401$ substituting ζ(s) value into the equation

$x^4 + y^4 = c.z^4$

$x^4 + y^4 = (7857/2401) \cdot z^4$

We use the general method

$\zeta(s) = r^n + s^n = c$

$s = \sqrt[n]{c - r^n}$

$\zeta(c) = 0$

$x = r \cdot z$

$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$

$n \rightarrow \infty$

$\zeta(s) = (27/21)^4 + (18/21)^4 = 636417/194481$

$= 7857/2401$

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 147) which is common factor by 21

$x = r \cdot z = (27/21)x147 = 189$

$y = s \cdot z = (18/21)x147 = 126$

ζ(s) value is not whole number then we can rewrite this equation

$x^4 + y^4 = (7857/2401) \cdot z^4$

$2401x^4 + 2401y^4 = 7857 \cdot z^4$

We verify that

$2401x^4 + 2401y^4 = 7857 \cdot z^4$

$2401 \times 189^4 + 2401 \times 126^4 = 7857 \cdot 147^4$

Sum left terms

$2401 \times 189^4 + 2401 \times 126^4 = 3668817358017$

Product right terms

$7857 \cdot 147^4 = 3668817358017$

Give another value of z (z = 252) which is common factor by 21

$x = r \cdot z = (27/21) \times 252 = 324$

$y = s \cdot z = (18/21) \times 252 = 216$

We verify that

$2401x^4 + 2401y^4 = 7857 \cdot z^4$

$2401 \times 324^4 + 2401 \times 216^4 = 7857 \cdot 252^4$

Sum left terms

$2401 \times 324^4 + 2401 \times 216^4 = 31685379731712$

Product right terms

$7857 \cdot 252^4 = 31685379731712$

Similarly we have many solutions of this equation

Solution 1 | Solution 2

$a = b = 2401, \quad c = 7857$

Solution 1	Solution 2
$x = 189$	$x = 324$
$y = 126$	$y = 216$
$z = 147$	$z = 252$

~~~~~/////~~~~~

zeta function ζ(s) is equal to c and c is not whole numbers

6) Find value a, b, c, x, y, z

$ax^5 + by^5 = cz^5$

Given $a = b$

~~~~~ Solve ~~~~~

First we divide both sides of equation $ax^5 + by^5 = cz^5$ by a

$x^5 + y^5 = (c/a) z^5$

Next we find $\zeta(s) = r^5 + s^5 = c$ and $\zeta(c) = 0$

Here c is equal to (c/a), get any faction of r and s we have

$\zeta(s) = (12/5)^5 + (8/5)^5 = 281600/3125$

$= 11264/125$

$\zeta(s) = 11264/125$ substituting $\zeta(s)$-value into the equation

$x^5 + y^5 = c.z^5$

$x^5 + y^5 = (11264/125)\cdot z^5$

We use the general method

$\zeta(s) = r^n + s^n = c$

$s = \sqrt[n]{c - r^n}$

$\zeta(c) = 0$

$x = r \cdot z$

$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$

$n \longrightarrow \infty$

$\zeta(s) = (12/5)^5 + (8/5)^5 = 281600/3125$

$= 11264/125$

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 75) which is common factor by 5

$x = r \cdot z = (12/5)x75 = 180$

$y = s \cdot z = (8/5)x75 = 120$

ζ(s) value is not whole number then we can rewrite this equation

$x^5 + y^5 = (11264/125) \cdot z^5$

$125x^5 + 125y^5 = 11264 \cdot z^5$

We verify that

$125x^5 + 125y^5 = 11264 \cdot z^5$

$125x180^5 + 125x120^5 = 11264 \cdot 75^5$

Sum left terms

$125x180^5 + 125x120^5 = 26730000000000$

Product right terms

$11264 \cdot z^5 = 26730000000000$

Give another value of z (z = 255) which is common factor by 5

$x = r \cdot z = (12/5)x255 = 612$

$y = s \cdot z = (8/5)x255 = 408$

We verify that

$125x^5 + 125y^5 = 11264 \cdot z^5$

$125x612^5 + 125x408^5 = 11264 \cdot 255^5$

Sum left terms

$125x612^5 + 125x408^5 = 12144888835200000$

Product right terms

$11264 \cdot 255^5 = 12144888835200000$

Similarly we have many solutions of this equation

Solution 1	Solution 2
a = b = 125, c = 11264	
x = 180	x = 612
y = 120	y = 408
z = 75	z = 255

~~~~~/////~~~~~

zeta function ζ(s) is equal to c and c is not whole numbers

7) Find value a, b, c, x, y, z

$ax^6 + by^6 = cz^6$

Given a = b

~~~~~ Solve ~~~~~

First we divide both sides of equation $ax^6 + by^6 = cz^6$ by a

$x^6 + y^6 = (c/a) z^6$

Next we find $\zeta(s) = r^6 + s^6 = c$ and $\zeta(c) = 0$

Here c is equal to (c/a), get any faction of r and s we have

$\zeta(s) = (9/7)^6 + (2/7)^6 = 531505/117649$

$\zeta(s) = 531505/117649$

substituting ζ(s)-value into the equation

$x^6 + y^6 = (c/a) z^6$

$x^6 + y^6 = (531505/117649) z^6$

We use the general method

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

$\zeta(s) = (9/7)^6 + (2/7)^6 = 531505/117649$

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 98) which is common factor by 7

$x = r \cdot z = (9/7) x 98 = 126$

$y = s \cdot z = (2/7) x 98 = 28$

ζ(s) value is not whole number then we can rewrite this equation

$x^6 + y^6 = (531505/117649) z^6$

$117649x^6 + 117649y^6 = 531505 \cdot z^6$

We verify that

$117649x^6 + 117649y^6 = 531505 \cdot z^6$

$117649x126^6 + 117649x28^6 = 531505 \cdot 98^6$

Sum left terms

$117649x126^6 + 117649x28^6 = 470829654641120320$

Product right terms

$531505 \cdot 98^6 = 470829654641120320$

Give another value of z (z = 196) which is common factor by 7

$x = r \cdot z = (9/7) x 196 = 252$

$y = s \cdot z = (2/7) x 196 = 56$

We verify that

$117649x^6 + 117649y^6 = 531505 \cdot z^6$

$117649x252^6 + 117649x56^6 = 531505 \cdot 196^6$

Sum left term

$117649x252^6 + 117649x56^6 = 30133097897031700480$

Product right term

$531505 \cdot 196^6 = 30133097897031700480$

Similarly we have many solutions of this equation

Solution 1

a = b = 117649, c = 531505

x = 126, y = 28, z = 98

Solution 2

x = 252, y = 56, z = 196

~~~~~/////~~~~~

zeta function ζ(s) is equal to c and c is not whole numbers

8) Find value x, y, z

$524288 \cdot x^7 + 524288 \cdot y^7 = 58401973 \cdot z^7$

~~~~~ Solve ~~~~~

First we divide both sides this equation by 524288

$524288 \cdot x^7 + 524288 \cdot y^7 = 58401973 \cdot z^7$

$x^7 + y^7 = (58401973/524288) \cdot z^7$

We have $\zeta(s) = r^7 + s^7 = 58401973/524288$

and $\zeta(58401973/524288) = 0$

Use hand calculator or PC for find value r and s, we see that

$\zeta(s) = r^7 + s^7 = 58401973/524288$

$\zeta(s) = (13/8)^7 + (15/8)^7 = 233607892/2097152$

$= 58401973/524288$

Here $\zeta(s)$ is equal to 58401973/524288, then c is not whole number

But we use this value for solution of equation $x^7 + y^7 = (58401973/524288) \cdot z^7$

Already use the general method

$$\zeta(s) = r^n + s^n = c$$

$$s = \sqrt[n]{c - r^n}$$

$$\zeta(c) = 0$$

$$x = r \cdot z$$

$$y = s \cdot z = \sqrt[n]{c - r^n} \cdot z$$

$$n \longrightarrow \infty$$

$\zeta(s) = (13/8)^7 + (15/8)^7 = 233607892/2097152$

$= 58401973/524288$

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 32) which is common factor by 8

$x = r \cdot z = (13/8)x32 = 52$

$y = s \cdot z = (15/8)x32 = 60$

Now we multiply both sides by 524288,

$524288 \cdot x^7 + 524288 \cdot y^7 = 58401973 \cdot z^7$

We verify that

$524288 \cdot x^7 + 524288 \cdot y^7 = 58401973 \cdot z^7$

$524288 \cdot 52^7 + 524288 \cdot 60^7 = 58401973 \cdot 32^7$

Sum left terms

$524288 \cdot 52^7 + 524288 \cdot 60^7 = 2006676512455000064$

Product right terms

$58401973 \cdot 32^7 = 2006676512455000064$

Similarly we have many solutions of this equation

Give another value of z (z = 56), which is common factor by

$x = r \cdot z = (13/8)x56 = 91$

$y = s \cdot z = (15/8)x56 = 105$

We verify that

$524288 \cdot x^7 + 524288 \cdot y^7 = 58401973 \cdot z^7$

$524288 \cdot 91^7 + 524288 \cdot 105^7 = 58401973 \cdot 56^7$

Sum left terms

$524288 \cdot 91^7 + 524288 \cdot 105^7 = 1008657467710405345 28$

Product right terms

$58401973 \cdot 56^7 = 100865746771040534528$

Similarly we have many solutions of this equation

$$524288 \cdot x^7 + 524288 \cdot y^7 = 58401973 \cdot z^7$$

Solution 1	Solution 2
$x = 91$	$x = 52$
$y = 105$	$y = 60$
$z = 56$	$z = 32$

Zeta function ζ(s) is equal to c and c is not whole numbers

9) Find value b, c, x, y, z

$$214358881 \cdot x^8 + b \cdot y^8 = c \cdot z^8$$

~~~~~ Solve ~~~~~

There is five unknowns, we have many method for solution of this equation $214358881 \cdot x^8 + b \cdot y^8 = c \cdot z^8$, also zeta function ζ(s) could help us to find the value of this equation, first get b = a = 214358881, next step find $\zeta(s) = r^8 + s^8 = c/\ 815730721 = c/11^8$ take any numerator of r and s. Use hand calculator or PC for find value r and s,

we have

$\zeta(s) = (3/11)^8 + (7/11)^8 = 5771362/214358881$ the equation above can rewrite

$$214358881 \cdot x^8 + 214358881 \cdot y^8 = 5771362 \cdot z^8$$

$$x^8 + y^8 = (5771362/214358881) \cdot z^8$$
~~~~~

We already used the general method

$\zeta(s) = (3/11)^8 + (7/11)^8 = 5771362/214358881$

$\zeta(5771362/214358881) = 0$

Here $\zeta(s)$ is equal to 5771362/214358881, then c is not whole number

But we use this value for solution of the equation

$x^8 + y^8 = (5771362/214358881) \cdot z^8$

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 121) which is common factor by 11

$x = r \cdot z = (3/11)x121= 33$

$y = s \cdot z = (7/11)x121= 77$

Now we multiply both sides by 214358881,

$214358881 \cdot x^8 + 214358881 \cdot y^8 = 5771362 \cdot z^8$

We verify that

$214358881 \cdot x^8 + 214358881 \cdot y^8 = 5771362 \cdot z^8$

$214358881 \cdot 33^8 + 214358881 \cdot 77^8 = 5771362 \cdot 121^8$

Sum left term

$214358881 \cdot 33^8 + 214358881 \cdot 77^8 = 265192524844885554253282$

Product right terms

$5771362 \cdot 121^8 = 265192524844885554253282$

Similarly we have many solutions of this equation

$214358881 \cdot x^8 + 214358881 \cdot y^8 = 5771362 \cdot z^8$

Take another numerator of r and s

we have

$\zeta(s) = (6/11)^8 + (8/11)^8 = 18456832/214358881$ the equation above can rewrite

$214358881 \cdot x^8 + 214358881 \cdot y^8 = 18456832 \cdot z^8$ or

$x^8 + y^8 = (18456832/214358881) \cdot z^8$

We already used the general method

$\zeta(s) = (6/11)^8 + (8/11)^8 = 18456832/214358881$

$\zeta(18456832/214358881) = 0$

Here $\zeta(s)$ is equal to 18456832/214358881, then c is not whole number

But we use this value for solution of the equation

$x^8 + y^8 = (18456832/214358881) \cdot z^8$

$x = r \cdot z$

$y = s \cdot z$

Give any value of z (z = 132) which is common factor by 11

$x = r \cdot z = (6/11)x132 = 72$

$y = s \cdot z = (8/11)x132 = 96$

Now we multiply both sides by 214358881,

$214358881 \cdot x^8 + 214358881 \cdot y^8 = 18456832 \cdot z^8$

We verify that

$214358881 \cdot x^8 + 214358881 \cdot y^8 = 18456832 \cdot z^8$

$214358881 \cdot 72^8 + 214358881 \cdot 96^8 = 18456832 \cdot 132^8$

Sum left terms

$214358881 \cdot 72^8 + 214358881 \cdot 96^8 = 170117349967306899534643 2$

Product right terms

$18456832 \cdot 132^8 = 1701173499673068995346432$

Similarly we have many solutions of this equation

$214358881 \cdot x^8 + 214358881 \cdot y^8 = 18456832 \cdot z^8$

Solution 1	Solution 2
b = a = 214358881	b = a = 214358881
c = 5771362	c = 18456832
x = 33	x = 72
y = 77	y = 96
z = 121	z = 132

~~~~~/////~~~~~
~~~~~

CHAPTER 9

The Diophantine Equations $x^n + b\cdot y^n = z^n$

~~~~~/////~~~~~

In this chapter we use zeta function ζ(s) is equal to 1 and zeta function ζ(1) is equal to 0 if n = 2. Invert n > 2, we use zeta function ζ(s) is equal to 1 and ζ(1) is not equal to 0. Similarly above we use new formula. Put ζ(s) $= \alpha^n + \beta^n = 1$

Now n > 2 if we find the values of α and β (0 < α, β <1) satisfying the equation $\alpha^n + \beta^n = 1$. Above we proved that $\alpha^n + \beta^n$ is not equal 1, also $\alpha^n + \beta^n \approx 1$, mean that

$\beta = \sqrt[n]{1 - \alpha^n}$ is irrational number, and we have formula

$$x = \alpha\cdot z$$

$$y = \sqrt[n]{1 - \alpha^n}\cdot z$$

$$0 < \alpha < 1$$

Before used this formula ,we must change this equation

$x^n + b\cdot y^n = z^n$ to form Fermat <u>$\boldsymbol{x^n + y^n = z^n}$</u> Or $x^n + (\sqrt[n]{b\cdot y})^n = z^n$

Here $y = \sqrt[n]{1 - \alpha^n}\ z$ is never whole number because the expression $\sqrt[n]{1 - \alpha^n}$ is irrational number, but in this equation y must equal to the whole number, therefore we find a general method for y is the whole number. In this case we reduce
~~~~~

expression $\sqrt[n]{1 - \alpha^n}$ by b, we have now y which is not dependent to the expression $\sqrt[n]{1 - \alpha^n}$, then y is the whole number, it 's easy by example following.

~~~~~/////~~~~~~

*) Find the value of x, y, z, such that

$$x^3 + 316 \cdot y^3 = z^3$$

~~~~~ solve ~~~~~

First we rewrite this equation

$$x^3 \quad + \quad 316 \cdot y^3 \quad = \quad z^3$$

To Fermat form $\boldsymbol{x^n} + \boldsymbol{y^n} = \boldsymbol{z^n}$

$$x^3 \quad + \quad (\sqrt[3]{1 - \alpha^3} \cdot y)^3 \quad = \quad z^3$$

second find zeta function ζ(s) is equal to 1 and ζ(1) is not equal to 0.

$\zeta(s) = \alpha^3 + \beta^3 = 1$, then the value of $\beta = \sqrt[3]{1 - \alpha^3}$

Find any value of α $(0 < \alpha < 1)$

for $\beta = \sqrt[3]{1 - \alpha^3} = \sqrt[3]{316} / ?$ $(0 < \beta < 1)$ therefore

α and β are the fractional number less than 1, use the hand-calculator, we have

$\alpha = 3/7$ because $\beta = \sqrt[3]{1 - \alpha^3} = \sqrt[3]{1 - (3/7)^3}$

$$\beta = \sqrt[3]{(343 - 27)} / 7 = \sqrt[3]{316} / 7$$

We already used the general method

$$x = \alpha \cdot z$$

$$y = \sqrt[3]{1 - \alpha^3} \cdot z$$

$$0 < \alpha < 1$$

Give any value of z (z = 63) which common factor by 7

$$x = \alpha.z = (3/7) \cdot 63 = 27$$

$$y = \sqrt[3]{1 - \alpha^3} \cdot z$$

$$y = \frac{\dfrac{\sqrt[3]{316} \times 63}{7}}{\sqrt[3]{316}} = \frac{63}{7} = 9$$

Substituting value of x, y, z, into this equation

$$x^3 + 316 \cdot y^3 = z^3$$

We verify that

$$27^3 + 316 \cdot 9^3 = 63^3$$

$$19683 + 230364 = 250047$$

Give another value of z (z = 84)

$$x = \alpha.z = (3/7) \cdot 84 = 36$$

$$y = \sqrt[3]{1 - \alpha^3} \cdot z$$

$$y = \frac{\frac{\sqrt[3]{316} \times 84}{7}}{\sqrt[3]{316}} = \frac{84}{7} = 12$$

Substituting value of x, y, z, into this equation

$$x^3 + 316 \cdot y^3 = z^3$$

We verify that

$$36^3 + 316 \cdot 12^3 = 84^3$$

$$46656 + 546048 = 592704$$

Solution 1

$x = 27, \quad y = 9, \quad z = 63$

Solution 2

$x = 36, \quad y = 12, \quad z = 84$

~~~~~/////~~~~~

*) Find the value of x, y, z, such that

$$x^4 + 2385 \cdot y^4 = z^4$$
~~~~~

~~~~~ solve ~~~~~

First we rewrite this equation

$x^4 + 2385 \cdot y^4 = z^4$

To Fermat form $\boldsymbol{x^n + y^n = z^n}$

$$x^4 + (\sqrt[4]{2385} \cdot y)^4 = z^4$$

second find zeta function ζ(s) is equal to 1 and ζ(1) is not equal to 0.

$\zeta(s) = \alpha^4 + \beta^4 = 1$, then the value of $\beta = \sqrt[4]{1 - \alpha^4}$

Find any value of α $(0 < \alpha < 1)$

for $\beta = \sqrt[4]{1 - \alpha^4} = \sqrt[4]{2385} / ?$ $(0 < \beta < 1)$ therefore

α and β are the fractional number less than 1, use the hand-calculator

$\alpha = 2/7$ because $\beta = \sqrt[4]{1 - \alpha^4} = \sqrt[4]{1 - (2/7)^4}$

$\beta = \sqrt[4]{2401 - 16} / 7 = \sqrt[4]{2385} / 7$

We already used the general method

$$x = \alpha \cdot z$$

$$y = \sqrt[4]{1 - \alpha^4} \cdot z$$
~~~~~

$$0 < \alpha < 1$$

Give any value of z (z = 119) which common factor by 7

$x = \alpha.z = (2/7) \cdot 119 = 34$

$y = \sqrt[4]{1 - \alpha^4} \cdot z$

$$y = \frac{\dfrac{\sqrt[4]{2385} \cdot 119}{7}}{\sqrt[4]{2385}} = \frac{119}{7} = 17$$

Substituting value of x, y, z, into this equation

$x^4 + 2385 \cdot y^4 = z^4$

We verify that

$34^4 + 2385 \cdot 17^4 = 119^4$

$1336336 + 199197585 = 200533921$

Give another value of z (z = 105)

$x = \alpha.z = (2/7) \cdot 105 = 30$

$y = \sqrt[4]{1 - \alpha^4} \cdot z$

$$y = \frac{\dfrac{\sqrt[4]{2385} \cdot 105}{7}}{\sqrt[4]{2385}} = \frac{105}{7} = 15$$

Substituting value of x, y, z, into this equation

$x^4 + 2385 \cdot y^4 = z^4$

We verify that

$30^4 + 2385 \cdot 15^4 = 105^4$

$810000 + 120740625 = 121550625$

Solution 1

$x = 34, \quad y = 17, \quad z = 119$

Solution 2

$x = 30, \quad y = 15, \quad z = 105$

~~~~~/////~~~~~

*) Find the value of x, y, z, such that

$$x^5 + 160027 \cdot y^5 = z^5$$

~~~~~ solve ~~~~~

First we rewrite this equation

$x^5 + 160027 \cdot y^5 = z^5$

To Fermat form $\boldsymbol{x^n + y^n = z^n}$

$$x^5 + (\sqrt[5]{160027} \cdot y)^5 = z^5$$

second find zeta function ζ(s) is equal to 1 and ζ(1) is not equal to 0.

$\zeta(s) = \alpha^5 + \beta^5 = 1$, then the value of $\beta = \sqrt[5]{1-\alpha^5}$

Find any value of α $(0 < \alpha < 1)$

for $\beta = \sqrt[5]{1-\alpha^5} = \sqrt[5]{160027}/?$ $(0 < \beta < 1)$ therefore

α and β are the fractional number less than 1,

$\alpha = 4/11$ because $\beta = \sqrt[5]{1-\alpha^5} = \sqrt[5]{1 - (4/11)^5}$

$\beta = \sqrt[5]{161051 - 1024}/11 = \sqrt[5]{160027}/11$

We already used the general method

$$x = \alpha \cdot z$$

$$y = \sqrt[5]{1-\alpha^5} \cdot z$$

$$0 < \alpha < 1$$

Give any value of z (z = 154) which common factor by 11

$x = \alpha.z = (4/11) \cdot 154 = 56$

$$y = \sqrt[5]{1 - \alpha^5} \cdot z$$

$$y = \frac{\dfrac{\sqrt[5]{160027} \times 154}{11}}{\sqrt[5]{160027}} = \frac{154}{11} = 14$$

Substituting value of x, y, z, into this equation

$x^5 + 160027 \cdot y^5 = z^5$

We verify that

$56^5 + 160027 \cdot 14^5 = 154^5$

550731776 + 86066361248 = 86617093024

Give another value of z (z = 132) which common factor by 11

$x = \alpha.z = (4/11) \cdot 132 = 48$

$$y = \sqrt[5]{1 - \alpha^5} \cdot z$$

$$y = \frac{\dfrac{\sqrt[5]{160027} \times 132}{11}}{\sqrt[5]{160027}} = \frac{132}{11} = 12$$

Substituting value of x, y, z, into this equation

$x^5 + 160027 \cdot y^5 = z^5$

We verify that

$48^5 + 160027 \cdot 12^5 = 132^5$

$254803968 + 39819838464 = 40074642432$

Solution 1

$x = 56, \quad y = 14, \quad z = 154$

Solution 2

$x = 48, \quad y = 12, \quad z = 132$

~~~~~/////~~~~~

*) Find the value of x, y, z, such that

$$x^6 + 1840825 \cdot y^6 = z^6$$

~~~~~ solve ~~~~~

We rewrite this equation

$x^6 + 1840825 \cdot y^6 = z^6$

To Fermat form $\boldsymbol{x^n + y^n = z^n}$

$$x^6 + (\sqrt[6]{1840825} \cdot y)^6 = z^6$$

Find zeta function ζ(s) is equal to 1 and ζ(1) is not equal to 0.

$\zeta(s) = \alpha^6 + \beta^6 = 1$, then the value of $\beta = \sqrt[6]{1-\alpha^6}$. is irrational number by proved above, where
$\zeta(s) = \alpha^6 + \beta^6$ is not equal to 1, ($\alpha^6 + \beta^6 \approx 1$) then
$\zeta(1)$ is not equal to 0

Find any value of α $(0 < \alpha < 1)$

for $\beta = \sqrt[6]{1-\alpha^6} = \sqrt[6]{1840825}/?$ because $(0 < \beta < 1)$ therefore

α and β are the fractional number less than 1,

$\alpha = 12/13$ because $\beta = \sqrt[6]{1-\alpha^6} = \sqrt[6]{1-(12/13)^6}$ then

$\beta = \sqrt[6]{1840825}/13$

We already used the general method

$$x = \alpha \cdot z$$

$$y = \sqrt[6]{1-\alpha^6} \cdot z$$

$$0 < \alpha < 1$$

Give any value of z (z = 208) which common factor by 13

$x = \alpha.z = (12/13) \cdot 208 = 192$

$$y = \sqrt[6]{1 - \alpha^6} \cdot z$$

$$y = \frac{\dfrac{\sqrt[6]{1840825} \times 208}{13}}{\sqrt[6]{1840825}} = \frac{208}{13} = 16$$

Substituting value of x, y, z, into this equation

$$x^6 + 1840825 \cdot y^6 = z^6$$

We verify that

$$192^6 + 1840825 \cdot 16^6 = 208^6$$

$$50096498540544 + 30883918643200 = 80980417183744$$

Give another value of z (z = 234) which common factor by 13

$$x = \alpha.z = (12/13) \cdot 234 = 216$$

$$y = \sqrt[6]{1 - \alpha^6} \cdot z$$

$$y = \frac{\dfrac{\sqrt[6]{1840825} \times 234}{13}}{\sqrt[6]{1840825}} = \frac{234}{13} = 18$$

Substituting value of x, y, z, into this equation

$$x^6 + 1840825 \cdot y^6 = z^6$$

We verify that

$216^6 + 1840825 \cdot 18^6 = 234^6$

101559956668416 + 62610552244800 = 164170508913216

Solution 1

$x = 192, \quad y = 16, \quad z = 208$

Solution 2

$x = 216, \quad y = 18, \quad z = 234$

~~~~~/////~~~~~

*) Find the value of x, y, z, such that

$$x^7 + 42664987 \cdot y^7 = z^7$$

~~~~~ solve ~~~~~

Similarly above we rewrite this equation

$x^7 + 42664987 \cdot y^7 = z^7$

To Fermat form $\boldsymbol{x^n + y^n = z^n}$

$$x^7 + (\sqrt[7]{42664987} \cdot y)^7 = z^7$$

Find zeta function ζ(s) is equal to 1 and ζ(1) is not equal to 0.

$\zeta(s) = \alpha^7 + \beta^7 = 1$, then the value of $\beta = \sqrt[7]{1-\alpha^7}$. is irrational number by proved above, where
$\zeta(s) = \alpha^7 + \beta^7$ is not equal to 1, $\alpha^7 + \beta^7 \approx 1$ then
$\zeta(1)$ is not equal to 0

Find any value of α $(0 < \alpha < 1)$

for $\beta = \sqrt[7]{1-\alpha^7} = \sqrt[7]{42664987} / ?$ because $(0 < \beta < 1)$ therefore

α and β are the fractional number less than 1,

$\alpha = 13/14$ because $\beta = \sqrt[7]{1-\alpha^7} = \sqrt[7]{1-(13/14)^7}$ then

$$\beta = \sqrt[7]{42664987} / 14$$

We already used the general method

$$x = \alpha \cdot z$$

$$y = \sqrt[7]{1-\alpha^7} \cdot z$$

$$0 < \alpha < 1$$

Give any value of z (z = 84) which common factor by 14

$$x = \alpha . z = (13/14) \cdot 84 = 78$$

$$y = \sqrt[7]{1-\alpha^7} \cdot z$$

$$y = \frac{\frac{\sqrt[7]{42664987}}{14} \times 84}{\sqrt[7]{42664987}} = \frac{84}{14} = 6$$

Substituting value of x, y, z, into this equation

$$x^7 + 42664987 \cdot y^7 = z^7$$

We verify that

$$78^7 + 42664987 \cdot 6^7 = 84^7$$

17565568854912 + 11943465800832 = 29509034655744

Give another value of z (z = 238) which common factor by 14

x = α.z = (13/14) · 238 = 221

$$y = \sqrt[7]{1 - \alpha^7} \cdot z$$

$$y = \frac{\frac{\sqrt[7]{42664987}}{14} \times 238}{\sqrt[7]{42664987}} = \frac{238}{14} = 17$$

Substituting value of x, y, z, into this equation

$$x^7 + 42664987 \cdot y^7 = z^7$$

We verify that

$$221^7 + 42664987 \cdot 17^7 = 238^7$$

25748143198497941 + 17507094149142251 = 43255237347640192

Solution 1

x = 78, y = 6, z = 84

Solution 2

x = 221, y = 17, z = 238

~~~~~/////~~~~~
~~~~~

CHAPTER 10

The Diophantine Equations $a \cdot x^n + b \cdot y^n = c \cdot z^n$

Coefficients and variable are the whole numbers with n approx to infinite ($n \to \infty$). Nearly many professors mathematical of the Universities of the World and mathematicians were researched about this, but we do not yet have a general method for solving these equations above. Here we could use zeta function ζ(s) is equal to 1 and ζ(1) is equal to 0 for solution of the equation $a \cdot x^2 + b \cdot y^2 = c \cdot z^2$ and zeta function ζ(s) is equal to 1 and ζ(1) is not equal to 0 for solution of the equation $a \cdot x^n + b \cdot y^n = c \cdot z^n$, $(n > 2)$ by the formula .

ζ(s) is equal to 1

ζ(1) is equal to 0

$$x = \alpha . z$$

$$y = \sqrt{1 - \alpha^2} .z$$

$$0 < \alpha < 1$$

and

ζ(s) is equal to 1

ζ(1) is not equal to 0

$$x = \alpha . z$$

$$y = \sqrt[n]{1 - \alpha^n} \, .z \quad n > 2$$

$$0 < \alpha < 1$$

Or

ζ(s) is equal to c

ζ(c) is equal to 0

$$x = \frac{\alpha \cdot z \sqrt[n]{c}}{\sqrt[n]{a}}$$

$$y = \frac{\sqrt[n]{1 - \alpha^n} \cdot z \sqrt[n]{c}}{\sqrt[n]{b}}$$

$$n \longrightarrow \infty$$

$$0 < \alpha < 1$$

~~~~~/////~~~~~

*) -Write an equation has form

$a \cdot x^4 + b \cdot y^4 = c \cdot z^4$

-Find the value x, y, z

~~~~~ solve ~~~~~

-Write an equation has form $a \cdot x^4 + b \cdot y^4 = c \cdot z^4$

We rewrite this equation to Fermat form $\boldsymbol{x^n + y^n = z^n}$

$(\sqrt[4]{a} \cdot x)^4 + (\sqrt[4]{b} \cdot y)^4 = (\sqrt[4]{c} \cdot z)^4$

Use the general method for solution of this Diophantine equation by zeta function ζ(s) is equal to 1 and ζ(1) is not equal to 0 then

$$\boldsymbol{x = \alpha \cdot z}$$

Therefore $\boldsymbol{x} = \sqrt[4]{a} \cdot x$ and $\boldsymbol{z} = (\sqrt[4]{c} \cdot z)$

$\sqrt[4]{a} \cdot x = \alpha \cdot (\sqrt[4]{c} \cdot z)$

$$x = \frac{\alpha . \sqrt[4]{c} \cdot z}{\sqrt[4]{a}}$$

and $\boldsymbol{y} = \sqrt[n]{1 - \alpha^n} \cdot z \quad n > 2$ then

$$y = \frac{\sqrt[4]{1 - \alpha^4} . \sqrt[4]{c} . z}{\sqrt[4]{b}}$$

We know that the variables x, y, z are the whole numbers therefore we must find values of the coefficients a, b, c for the variables are the whole numbers . In case we have many values of a, b, c, (27, 592, 432 ….)

Substituting the values of the coefficients a, b, c above. Hence there is a equation also we want to write following

$$27 \cdot x^4 + 592 \cdot y^4 = 432 \cdot z^4$$

-Find the value of x, y, z

We now rewrite this equation $27 \cdot x^4 + 592 \cdot y^4 = 432 \cdot z^4$ to Fermat's form $\boldsymbol{x^n + y^n = z^n}$

$(\sqrt[4]{a} \cdot x)^4 + (\sqrt[4]{b} \cdot y)^4 = (\sqrt[4]{c} \cdot z)^4$

$(\sqrt[4]{27} \cdot x)^4 + (\sqrt[4]{592} \cdot y)^4 = (\sqrt[4]{432} \cdot z)^4$

Multiply both sides of the equation by 3 we have

$(\sqrt[4]{81} \cdot x)^4 + (\sqrt[4]{1776} \cdot y)^4 = (\sqrt[4]{1296} \cdot z)^4$

$a = \sqrt[4]{81} = 3$

$c = \sqrt[4]{1296} = 6$

Next step find the value of the variables x, y, z

$$x = \frac{\alpha \cdot \sqrt[4]{c} \cdot z}{\sqrt[4]{a}}$$

We do not yet know the value of α, but from y below there will be β-value

$$y = \frac{\sqrt[4]{1-\alpha^4} \cdot \sqrt[4]{c} \cdot z}{\sqrt[4]{b}}$$

$\sqrt[4]{b}$-value must reduce irrational number of the numerator of β-value . (numerator of $\beta = \sqrt[4]{1-\alpha^4} = \sqrt[4]{1776}$) for y is the whole number, hence α = 5/7 , give any value of z (z = 28) which is common factor by 7, substituting the values above into x

$$x = \frac{\alpha \cdot \sqrt[4]{c} \cdot z}{\sqrt[4]{a}}$$

$$x = \frac{\frac{5}{7} \cdot 6 \cdot 28}{3} = 40$$

And similarly above we have value of y

$$y = \frac{\sqrt[4]{1-\alpha^4} \cdot \sqrt[4]{c} \cdot z}{\sqrt[4]{b}}$$

$$y = \frac{\frac{\sqrt[4]{1776}}{7} \cdot 6 \cdot 28}{\sqrt[4]{1776}} = 24$$

We verify for sure

Substituting the values of x, y, z, into the equation

$(\sqrt[4]{27}\cdot x)^4 + (\sqrt[4]{592}\cdot y)^4 = (\sqrt[4]{432}\cdot z)^4$

$27\cdot x^4 + 592\cdot y^4 = 432\cdot z^4$

$27 \cdot 40^4 + 592 \cdot 24^4 = 432 \cdot 28^4$

69120000+196411392 = 265531392

Solution

a = 27, b = 592, c = 432

x = 40, y = 24, z = 28

Similarly above we have many values of this equation

~~~~~/////~~~~~

*)- Write an equation has form

$a\cdot x^5 + b\cdot y^5 = c\cdot z^5$

-Find the value x, y, z

~~~~~ solve ~~~~~

- Write an equation has form

We rewrite this equation to Fermat form $\boldsymbol{x^n + y^n = z^n}$

$(\sqrt[5]{a} \cdot x)^5 + (\sqrt[5]{b} \cdot y)^5 = (\sqrt[5]{c} \cdot z)^5$

Use the general method

$$x = \alpha . z$$

Here $x = \sqrt[5]{a} \cdot x$ and

$z = (\sqrt[5]{c} \cdot z)$ therefore

$$\sqrt[5]{a} \cdot x = \alpha \cdot (\sqrt[5]{c} \cdot z)$$

$$x = \frac{\alpha \cdot \sqrt[5]{c} \cdot z}{\sqrt[5]{a}}$$

also we have $y = \sqrt[n]{1 - \alpha^n} \cdot z \quad n > 2$ then

$$y = \frac{\sqrt[5]{1 - \alpha^5} \cdot \sqrt[5]{c} \cdot z}{\sqrt[5]{b}}$$

We know that the variables x, y, z are the whole numbers therefore we must find values of the coefficients a, b, c for the variables are the whole numbers . In case we have many values of a, b, c, (2, 4202, 6250 ….)

Substituting the values of the coefficients a, b, c above. We have the equation

$$2 \cdot x^5 + 4202 \cdot y^5 = 6250 \cdot z^5$$

-Find the value of x, y, z

Rewrite this equation $2 \cdot x^5 + 4202 \cdot y^5 = 6250 \cdot z^5$

to Fermat's form $\boldsymbol{x^n + y^n = z^n}$

$(\sqrt[5]{a} \cdot x)^5 + (\sqrt[5]{b} \cdot y)^5 = (\sqrt[5]{c} \cdot z)^5$

we have $(\sqrt[5]{2} \cdot x)^5 + (\sqrt[5]{4202} \cdot y)^5 = (\sqrt[5]{6250} \cdot z)^5$

Or $(\sqrt[5]{2} \cdot x)^5 + (\sqrt[5]{2} \cdot \sqrt[5]{2101}\, y)^5 = (5\sqrt[5]{2} \cdot z)^5$

In other way we divide both sides of this equation $2 \cdot x^5 + 4202 \cdot y^5 = 6250 \cdot z^5$ by 2 we have

$x^5 + 2101 \cdot y^5 = 3125 \cdot z^5$

Also we must rewrite to Fermat's form $\boldsymbol{x^n + y^n = z^n}$

$x^5 + (\sqrt[5]{2101} \cdot y)^5 = (5 \cdot z)^5$

From this equation we have a = 1

b = 2101 hence we easy find β-value . (numerator of β = $\sqrt[5]{2101}$) for y is the whole number, therefore α = 4/5

c = 5

Next step find the value of the variables x, y, z

Give any value of z (z = 25) which is common factor by 5, substituting the values above into x

$$x = \frac{\alpha \cdot \sqrt[5]{c} \cdot z}{\sqrt[5]{a}}$$

x = (4/5) · 5 ·25 = 100

And

$$y = \frac{\sqrt[5]{1-\alpha^{5}} \cdot \sqrt[5]{c} \cdot z}{\sqrt[5]{b}}$$

$$y = \frac{\dfrac{\sqrt[5]{2101}}{5} \cdot 5 \cdot 25}{\sqrt[5]{2101}} = 25$$

We verify for sure

Substituting the values of x, y, z, into the equation

$2 \cdot x^5 + 4202 \cdot y^5 = 6250 \cdot z^5$

$2 \cdot 100^5 + 4202 \cdot 25^5 = 6250 \cdot 25^5$

20000000000+41035156250 = 61035156250

Solution

a = 2, b = 4202, c = 6250

x = 100, y = 25, z = 25

Similarly above we have many values of this equation

~~~~~/////~~~~~
~~~~~

*) -Write an equation has form

$a \cdot x^6 + b \cdot y^6 = c \cdot z^6$

-Find the value x, y, z

Give $\alpha = 7/13$

~~~~~ solve ~~~~~

-Write an equation has form

We now rewrite this equation $a \cdot x^6 + b \cdot y^6 = c \cdot z^6$

to Fermat's form $\boldsymbol{x^n + y^n = z^n}$

We have $(\sqrt[6]{a} \cdot x)^6 + (\sqrt[6]{b} \cdot y)^6 = (\sqrt[6]{c} \cdot z)^6$

Use the general method

$$\boldsymbol{x = \alpha . z}$$

Therefore $\sqrt[6]{a} \cdot x = \alpha \cdot (\sqrt[6]{c} \cdot z)$ or

$$x = \frac{\alpha \cdot \sqrt[6]{c} \cdot z}{\sqrt[6]{a}}$$

and $\boldsymbol{y} = \sqrt[n]{1 - \alpha^n} \cdot z$ $n > 2$ then
~~~~~

$$y = \frac{\sqrt[6]{1 - \alpha^6} \cdot \sqrt[6]{c} \cdot z}{\sqrt[6]{b}}$$

We know that the variables x, y, z are the whole numbers therefore we must find values of the coefficients a, b, c ?

We have $\alpha = 7/13$ then

$$\beta = \frac{\sqrt[6]{1 - \alpha^6}}{13} = \frac{\sqrt[6]{13^6 - 7^6}}{13} = \frac{\sqrt[6]{4709160}}{13}$$

Therefore $\sqrt[6]{b} = \sqrt[6]{4709160}$ or $b = 4709160$

For easy we get $a = 1$ and $c = 64 = 2^6$

Substituting the values of the coefficients a, b, c above into the equation we have

$a \cdot x^6 + b \cdot y^6 = c \cdot z^6$

$x^6 + 4709160 \cdot y^6 = 64 \cdot z^6$

Find the value of x, y, z

Rewrite this equation $x^6 + 4709160 \cdot y^6 = 64 \cdot z^6$

to Fermat's form $\boldsymbol{x^n + y^n = z^n}$

Or $(\sqrt[6]{a} \cdot x)^6 + (\sqrt[6]{b} \cdot y)^6 = (\sqrt[6]{c} \cdot z)^6$

We have $x^6 + \sqrt[6]{4709160} \cdot y^6 = (2 \cdot z)^6$

Give any value of z (z = 39…) which common factor by 13

Be back

$$x = \frac{\alpha \cdot \sqrt[6]{c} \cdot z}{\sqrt[6]{a}}$$

$$x = \frac{2 \cdot 39 \cdot 7}{13} = 42$$

And

$$y = \frac{\sqrt[6]{1-\alpha^6} \cdot \sqrt[6]{c} \cdot z}{\sqrt[6]{b}}$$

$$y = \frac{\frac{\sqrt[6]{4709160}}{13} \cdot 2 \cdot 39}{\sqrt[6]{4709160}} = 6$$

We verify for sure

Substituting the values of x, y, z, into the equation

$x^6 + 4709160 \cdot y^6 = 64 \cdot z^6$

$42^6 + 4709160 \cdot 6^6 = 64 \cdot 39^6$

5489031744+ 219710568960 = 225199600704

Solution

a = 1, b = 4709160, c = 64

x = 42, y = 6, z = 39

Similarly above we have many values of this equation

~~~~~/////~~~~~

*) -Write an equation has form

$a \cdot x^{13} + b \cdot y^{13} = c \cdot z^{13}$

-Find the value x, y, z

Given that α = 3/4 and α = 8/9

~~~~~ solve ~~~~~

-Write an equation has form:

$a \cdot x^{13} + b \cdot y^{13} = c \cdot z^{13}$

We rewrite this equation to Fermat form $\boldsymbol{x^n + y^n = z^n}$

$(\sqrt[13]{a} \cdot x)^{13} + (\sqrt[13]{b} \cdot y)^{13} = (\sqrt[13]{c} \cdot z)^{13}$

Use the general method for solution of this Diophantine equation by zeta function ζ(s) is equal to 1 and ζ(1) is not equal to 0 then

$$x = \alpha \cdot z$$

Therefore $x = \sqrt[13]{a} \cdot x$ and $z = (\sqrt[13]{c} \cdot z)$

$$\sqrt[13]{a} \cdot x = \alpha \cdot (\sqrt[13]{c} \cdot z)$$

$$x = \frac{\alpha \cdot \sqrt[13]{c} \cdot z}{\sqrt[13]{a}}$$

and $y = \sqrt[n]{1 - \alpha^n} \cdot z$ $n > 2$ then

$$y = \frac{\sqrt[13]{1 - \alpha^{13}} \cdot \sqrt[13]{c} \cdot z}{\sqrt[13]{b}}$$

a) $\alpha = 3/4$

We know that the variables x, y, z are the whole numbers therefore we must find values of the coefficients a, b, c for the variables are the whole numbers . In case we choose any values of a, c, for x and z are integers example $a = 2^{13}, 3^{13}, 4^{13}$...and $c = 7^{13}, 8^{13}, 9^{13}$, … the value of b is only equal to 65514541 = $4^{13} - 3^{13}$, because $\alpha = 3/4$

Substituting the values of the coefficients a, b, c above. Hence there is a equation also we want to write following

$a \cdot x^{13} + b \cdot y^{13} = c \cdot z^{13}$

$8192 \cdot x^{13} + 65514541 \cdot y^{13} = 96889010407 \cdot z^{13}$

-Find the value of x, y, z

We now rewrite this equation

$8192 \cdot x^{13} + 65514541 \cdot y^{13} = 96889010407 \cdot z^{13}$

to Fermat's form $\boldsymbol{x^n + y^n = z^n}$

$(2 \cdot x)^{13} + \sqrt[13]{65514541} \cdot y^{13} = (7 \cdot z)^{13}$

$a = 2^{13}$

$c = 7^{13}$

Next step find the value of the variables x, y, z, here we choose any value of z (…24, 28, 32 …)which is common factor by 4

$$x = \frac{\alpha \cdot \sqrt[13]{c} \cdot z}{\sqrt[13]{a}}$$

$$x = \frac{\frac{3}{4} \cdot 7 \cdot 24}{2} = 63$$

Similarly above

$$y = \frac{\sqrt[13]{1-\alpha^{13}} \cdot \sqrt[13]{c} \cdot z}{\sqrt[13]{b}}$$

$$y = \frac{\frac{\sqrt[13]{65514541}}{4} \cdot 7 \cdot 24}{\sqrt[13]{65514541}} = 42$$

We verify for sure

Substituting the values of x, y, z, into the equation

$8192 \cdot x^{13} + 65514541 \cdot y^{13} = 96889010407 \cdot z^{13}$

$8192 \cdot 63^{13} + 65514541 \cdot 42^{13} = 96889010407 \cdot 24^{13}$

Sum left side

$8192 \cdot 63^{13} + 65514541 \cdot 42^{13} =$

84922087747184192618514874368

Product right side

$96889010407 \cdot 24^{13} = 84922087747184192618514874368$

b) $\alpha = 8/9$

We know that the variables x, y, z are the whole numbers therefore we must find values of the coefficients a, b, c for the variables are the whole numbers . In case we choose any values of a, c, for x and z are integers example $a = 3^{13}$, 4^{13}...and $c = 8^{13}$,

9^{13}, … the value of b is only equal to 1992110014441 = $9^{13} - 8^{13}$, because $\alpha = 8/9$

Substituting the values of the coefficients a, b, c above, we have equation:

$$a \cdot x^{13} + b \cdot y^{13} = c \cdot z^{13}$$

$$1594323 \cdot x^{13} + 1992110014441 \cdot y^{13} = 549755813888 \cdot z^{13}$$

-Find the value of x, y, z

We now rewrite this equation

$$1594323 \cdot x^{13} + 1992110014441 \cdot y^{13} = 549755813888 \cdot z^{13}$$

to Fermat's form $\boldsymbol{x^n + y^n = z^n}$

$$(3 \cdot x)^{13} + \sqrt[13]{(1992110014441)} \cdot y^{13} = (8 \cdot z)^{13}$$

$a = 3^{13}$

$c = 8^{13}$

Next step find the value of the variables x, y, z, here we choose any value of z (729…) which is common factor by 9

$$x = \frac{\alpha \cdot \sqrt[13]{c} \cdot z}{\sqrt[13]{a}}$$

$$x = \frac{\frac{8}{9} \cdot 8 \cdot 729}{3} = 1728$$

Similarly above we have

$$y = \frac{\sqrt[13]{1-\alpha^{13}} \cdot \sqrt[13]{c} \cdot z}{\sqrt[13]{b}}$$

$$y = \frac{\frac{\sqrt[13]{1992110014441}}{9} \cdot 8 \cdot 729}{\sqrt[13]{1992110014441}} = 648$$

We verify for sure

Substituting the values of x, y, z, into the equation

$1594323 \cdot x^{13} + 1992110014441 \cdot y^{13} = 549755813888 \cdot z^{13}$

$1594323 \cdot 1728^{13} + 1992110014441 \cdot 648^{13} = 549755813888 \cdot 729^{13}$

Sum left side

$1594323 \cdot 1728^{13} + 1992110014441 \cdot 648^{13} =$

9.0287514793906997173120179008158e+48

Product right side

$549755813888 \cdot 729^{13} =$ 9.0287514793906997173120179008158e+48

Solution 1

a = 8192, b = 65514541, c = 65514541

x = 63, y = 42, z = 24

Solution 2

a = 1594323, b = 1992110014441, c = 549755813888

x = 1728, y = 648, z = 729

Similarly above we have many values of this equation

~~~~~/////~~~~~

Exercises

1*) Find the value of x, y, z, such that

$$x^3 + 218\cdot y^3 = z^3$$

2*) Find the value of x, y, z, such that

$$x^3 + 631\cdot y^3 = z^3$$

3*) Find the value of x, y, z, such that

$$x^3 + 665\cdot y^3 = z^3$$

4*) Find the value of x, y, z, such that

$$x^3 + 988\cdot y^3 = z^3$$

5*) Find the value of b, x, y, z, such that

$$x^3 + b\cdot y^3 = z^3$$

Given that $\alpha = 8/9$
~~~~~

6*) Find the value of b, x, y, z, such that

$$x^3 + b\cdot y^3 = z^3$$

Given that $\alpha = 12/17$

7*) Find the value of b, x, y, z, such that

$$x^3 + b\cdot y^3 = z^3$$

Given that $\alpha = 13/15$

8*) Find the value of b, x, y, z, such that

$$x^4 + b\cdot y^4 = z^4$$

Given that $\alpha = 5/9$

9*) Find the value of b, x, y, z, such that

$$x^4 + b\cdot y^4 = z^4$$

Given that $\alpha = 7/11$

10*) Find the value of b, y, z, such that

$$x^4 + b\cdot y^4 = z^4$$

Given that $x = 28$

11*) Find the value of b, x, y, such that

$$x^4 + b\cdot y^4 = z^4$$

Given that $z = 529$

12*) Find the value of b, x, y, z, such that

$$x^4 + b \cdot y^4 = z^4$$

Given that α = 17/21

13*) Find the value of b, x, y, z, such that

$$x^5 + b \cdot y^5 = z^5$$

Given that α = 5/6

14*) Find the value of b, x, y, z, such that

$$x^5 + b \cdot y^5 = z^5$$

Given that α = 37/42

15*) Find the value of b, x, z, such that

$$x^5 + b \cdot y^5 = z^5$$

Given that y = 39

16*) Find the value of b, x, y, z, such that

$$x^6 + b \cdot y^6 = z^6$$

Given that α = 11/16

17*) Find the value of a, x, y, z, such that

$$a \cdot x^6 + y^6 = z^6$$

Given that α = 13/16

18*) Find the value of a, x, y, such that

$$a \cdot x^6 + y^6 = z^6$$

Given that z = 49

19*) Find the value of a, x, y, z, such that

$$a \cdot x^7 + y^7 = z^7$$

Given that α = 13/17

20*) Find the value of a, x, y, z, such that

$$a \cdot x^7 + y^7 = z^7$$

Given that α = 3/11

21*) Find the value of b, x, y, z, such that

$$x^8 + b \cdot y^8 = z^8$$

Given that α = 3/17

22*) Find the value of b, x, y, z, such that

$$x^{12} + b \cdot y^{12} = z^{12}$$

Given that α = 9/12

23*) Find the value of x, y, z, such that

$$x^{15} + b \cdot y^{15} = z^{15}$$

Given that b = 4277376525367

24*) Find the value of b, y, z, such that

$$x^{57} + b\cdot y^{57} = z^{57}$$

Given that x = 432

25*) Find the value of b, x, y, such that

$$x^{537} + b\cdot y^{537} = z^{537}$$

Given that z = 4323

26*) Find the value of b, x, y, such that

$$x^{2004} + b\cdot y^{2004} = z^{2004}$$

Given that z = 7326

27**) Find the value of t, u, v, x, y, z, such that

$$17716055191287\cdot t^{14} + u^{14} + v^{14} + x^{14} + y^{14} = z^{14}$$

Give α = 7/9

28**) Find the value of t, u, v, x, y, z, such that

$$t^{17} + u^{17} + v^{17} + x^{17} + 214923606079999 \cdot y^{17} = z^{17}$$

Give α = 5/7

29*)- Write an equation has form

$$a\cdot x^{14} + b\cdot y^{14} = c\cdot z^{14}$$

-Find the value x, y, z

30*)- Write an equation has form

$a \cdot x^{14} + b \cdot y^{14} = c \cdot z^{14}$

-Find the value x, y, z

Given that $\alpha = 4/7$

31*)- Write an equation has form

$a \cdot x^{15} + b \cdot y^{15} = c \cdot z^{15}$

-Find the value x, y, z

Given that $\alpha = 6/7$

32*)- Write an equation has form

$a \cdot x^{16} + b \cdot y^{16} = c \cdot z^{16}$

-Find the value x, y, z

Given that $\alpha = 11/13$

33*)- Write an equation has form

$a \cdot x^{18} + b \cdot y^{18} = c \cdot z^{18}$

-Find the value x, y, z

Given that $\alpha = 3/7$

34*)- Write an equation has form

$a \cdot x^{24} + b \cdot y^{24} = c \cdot z^{24}$

-Find the value x, y, z

Given that $\alpha = 23/27$

35***)- Write an equation has form

$a \cdot A^{x} + b \cdot B^{x} = c \cdot C^{x}$

For the value of A, B, C do not change, when the exponent x approximate infinitely $(x \rightarrow \infty)$

-Find the value A, B, C

-verify for sure

36***)- Write an equation has form

$a \cdot A^{x} + b \cdot B^{x} = c \cdot C^{x}$

For the value of A, B, C do not change, when the exponent x approximate infinitely $(x \rightarrow \infty)$

-Find the value A, B, C

-verify for sure

Given that $a = c = 1$

37***)- Write an equation has form

$a \cdot A^{x} + b \cdot B^{x} = c \cdot C^{x}$

For the value of A, B, C do not change, when the exponent x approximate infinitely ($x \to \infty$)

-Find the value A, B, C

-verify for sure
Given that $\alpha = 3/7$

38***)- Write an equation has form

$$a \cdot A^x + b \cdot B^x = c \cdot C^x$$

For the value of A, B, C do not change, when the exponent x approximate infinitely ($x \to \infty$)

-Find the value A, B, C

-verify for sure
Given that $\alpha = 7/13$

39***)- Write an equation has form

$$a \cdot A^x + b \cdot B^x = c \cdot C^x$$

For the value of A, B, C do not change, when the exponent x approximate infinitely ($x \to \infty$)

-Find the value A, B, C

-verify for sure
Give C = 16169

40***)- Write an equation has form

$a \cdot A^x + b \cdot B^x = c \cdot C^x$

For the value of A, B, C do not change, when the exponent x approximate infinitely ($x \rightarrow \infty$)

-Find the value A, B, C

-verify for sure
Given that A = 7667

41***)- Write an equation has form

$a \cdot A^x + b \cdot B^x = c \cdot C^x$

For the value of A, B, C do not change, when the exponent x approximate infinitely ($x \rightarrow \infty$)

-Find the value A, B, C

-verify for sure
Given that B = 3453

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CHAPTER 11

Complex Diophantine Equations

The Diophantine equation with form of

$$a\cdot v^n + b\cdot x^n + c\cdot y^n = d\cdot z^n$$

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1) The Diophantine equation with form: $a \cdot v^8 + b\cdot x^8 + c\cdot y^8 = d\cdot z^8$

1) Write Diophantine equation with form below

$a\cdot v^8 + b\cdot x^8 + c\cdot y^8 = d\cdot z^8$

2) Find the value of v, x, y, z

~~~~~ Solve ~~~~~

1) Write Diophantine equation

We find coefficients a, b, c, d of the equation $a\cdot v^8 + b\cdot x^8 + c\cdot y^8 = d\cdot z^8$ We have many method for solution of this Diophantine equation. Also zeta function ζ(s) could help us to find the value of this equation, first find ζ(s) and ζ(s) is not whole number. Take any fractional numbers of r and s, example : $\zeta(s) = (7/12)^8 + (5/12)^8$ Similarly above we have solution of coefficients of a, b, c, d following

a = 429981696,

b = 415882107

c = 2149

d = 6155426

we have equation

$$429981696 \cdot v^8 + 415882107 \cdot x^8 + 2149 \cdot y^8 = 6155426 \cdot z^8$$

2) Find the value of v, x, y, z of Diophantine equation

With my general method above, we can find the value of v, x, y, z following

v = 49

x = 35

y = 105

z = 84

Substituting value of v, x, y, z, into this equation

We verify that

$$429981696 \cdot 49^8 + 415882107 \cdot 35^8 + 2149 \cdot 105^8 = 6155426 \cdot 84^8$$

Sum left terms

$$429981696 \cdot 49^8 + 415882107 \cdot 35^8 + 2149 \cdot 105^8$$

$$= 15257817049008884023296$$

Product right terms

$6155426 \cdot 84^8 \quad = \quad 1525781704900888402329 6$

Solution

$429981696 \cdot v^8 + 415882107 \cdot x^8 + 2149 \cdot y^8 = 6155426 \cdot z^8$

a = 429981696,	v = 49
b = 415882107	x = 35
c = 2149	y = 105
d = 6155426	z = 84

~~~~~/////~~~~~

2) The Diophantine equation with form: $a \cdot v^8 + b \cdot x^8 + c \cdot y^8 = d \cdot z^8$

1) Write Diophantine equation with form below

$a \cdot v^8 + b \cdot x^8 + c \cdot y^8 = d \cdot z^8$

2) Find the value of v, x, y, z

~~~~~ Solve ~~~~~

1) Write Diophantine equation

We find coefficients a, b, c, d of the equation $a \cdot v^8 + b \cdot x^8 + c \cdot y^8 = d \cdot z^8$

Take any value of r and s Example : $\zeta(s) = (3/11)^8 + (9/11)^8$

We already used the general method

Similarly above we have solution of coefficients of a, b, c, d following

a = 214358881

b = 9671381

c = 524

d = 43053282

And we have Diophantine equation

$214358881 \cdot v^8 + 9671381 \cdot x^8 + 524 \cdot y^8 = 43053282 \cdot z^8$

2) Find the value of v, x, y, z of Diophantine equation

With my general method above, we can find the value of v, x, y, z following

v = 42, x = 126, y = 630, z = 154

Substituting value of v, x, y, z, into this equation

We verify that

$214358881 \cdot 42^8 + 9671381 \cdot 126^8 + 524 \cdot 630^8 = 43053282 \cdot 154^8$

Sum left terms

$214358881 \cdot 42^8 + 9671381 \cdot 126^8 + 524 \cdot 630^8$

$= 1361984077763495079399475 2$

Product right terms

$43053282 \cdot 154^8 \quad = \quad 136198407776349507939947 52$

Solution

$214358881 \cdot v^8 + 9671381 \cdot x^8 + 126 \cdot y^8 = 43053282 \cdot z^8$

$a = 214358881,$	$v = 42$
$b = 9671381$	$x = 126$
$c = 126$	$y = 1630$
$d = 43053282$	$z = 154$

~~~~~/////~~~~~

3) The Diophantine equation with form

$$a \cdot u^n + b \cdot v^n + c \cdot x^n + d \cdot y^n = e \cdot z^n$$

There is ten unknowns, include 5 variables and 5 coefficients. We have many methods for solution of this equation

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1) Write Diophantine equation with form below

$a \cdot u^9 + b \cdot v^9 + c \cdot x^9 + d \cdot y^9 = e \cdot z^9$

2) Find the value of u, v, x, y, z

~~~~~ Solve ~~~~~

1) Write Diophantine equation

Now we use zeta function ζ(s) is equal to c and ζ(c) is equal to 0, here ζ(c) is not whole number.

First we find the value of ζ(s) for the equation below

Take any value of zeta function ζ(s) is equal to c and ζ(c) is equal to 0,

Example: $\zeta(s) = (4/9)^9 + (7/9)^9$

Now we find value of coefficients for the equation

$$a\cdot u^9 + b\cdot v^9 + c\cdot x^9 + d\cdot y^9 = e\cdot z^9$$

Similarly above step by step, we have solutions

a = 17258, 387420489

b = 47731275,

c = 225153,

d = 272142153,

e = 40615751,

we have equation

$$17258\cdot u^9 + 47731275\cdot v^9 + 225153\cdot x^9 + 272142153\cdot y^9 = 40615751\cdot z^9$$
~~~~~

2) Find the value of u, v, x, y, z

We find value of variables, similarly we use zeta function ζ(s) is equal to c and ζ(c) is equal to 0, here ζ(c) is not whole number above we have

u = 156

v = 52

x = 182

y = 91

z = 117

Substituting value of u, v, x, y, z, into this equation

$17258\cdot u^9 + 47731275\cdot v^9 + 225153\cdot x^9 + 272142153\cdot y^9 = 40615751\cdot z^9$

We verify that

$17258\cdot 156^9 + 47731275\cdot 52^9 + 225153\cdot 182^9 + 272142153\cdot 91^9 =$

$40615751\cdot 117^9$

Sum left terms

$17258\cdot 156^9 + 47731275\cdot 52^9 + 225153\cdot 182^9 + 272142153\cdot 91^9 =$

9443040446061904716103688+132688452205934169292800+

4933103528377459205216409 6 +
11645773714018029760398888 3

=

16686576492076701429705614 7

Product right terms

$40615751 \cdot 117^9$ =
16686576492076701429705614 7

Similarly we have many solutions of this equation

$17258 \cdot u^9 + 47731275 \cdot v^9 + 225153 \cdot x^9 + 272142153 \cdot y^9 = 40615751 \cdot z^9$

Solution 1

a = 17258, u = 156

b = 47731275, v = 52

c = 225153, x = 182

d = 272142153, y = 91

e = 40615751, z = 117

~~~~~/////~~~~~

4) The Diophantine equation with form

$$a \cdot t^n + b \cdot u^n + c \cdot v^n + d \cdot x^n + e \cdot y^n = f \cdot z^n$$

~~~~~/////~~~~~

1) Write equation which has the form below

$a\cdot t^{10} + b\cdot u^{10} + c\cdot v^{10} + d\cdot x^{10} + e\cdot y^{10} = f\cdot z^{10}$

Coefficients a, b, c, d, e, f, are differently

2)Find the value of variables of this equation

~~~~~ Solve ~~~~~

1) Write equation which has the form below

$a\cdot t^{10} + b\cdot u^{10} + c\cdot v^{10} + d\cdot x^{10} + e\cdot y^{10} = f\cdot z^{10}$

First we find the value of ζ(s) for the equation below

$a\cdot t^{10} + b\cdot u^{10} + c\cdot v^{10} + d\cdot x^{10} + e\cdot y^{10} = f\cdot z^{10}$

Second similarly above we have solutions of coefficients

a = 4223737,

b = 5412,

c = 2443549,

d = 124,

e = 28,

f = 88745

Substituting value of coefficients into the equation

$a\cdot t^{10} + b\cdot u^{10} + c\cdot v^{10} + d\cdot x^{10} + e\cdot y^{10} = f\cdot z^{10}$
~~~~~

we have equation

$4223737 \cdot t^{10} + 5412 \cdot u^{10} + 2443549 \cdot v^{10} + 124 \cdot x^{10} + 28 \cdot y^{10} = 88745 \cdot z^{10}$

2)Find the value of variables of this equation

Find value of variables by zeta function ζ(s) is equal to c and ζ(c) is equal to 0, similarly above we have

t = 26

u = 52

v = 39

x = 117

y = 130

z = 65

Substituting value of t, u, v, x, y, z, into this equation

$4223737 \cdot t^{10} + 5412 \cdot u^{10} + 2443549 \cdot v^{10} + 124 \cdot x^{10} + 28 \cdot y^{10} = 88745 \cdot z^{10}$

We verify that

$4223737 \cdot 26^{10} + 5412 \cdot 52^{10} + 2443549 \cdot 39^{10} + 124 \cdot 117^{10} + 28 \cdot 130^{10} =$

$88745 \cdot 65^{10}$

Sum left term

$4223737 \cdot 26^{10} + 5412 \cdot 52^{10} + 2443549 \cdot 39^{10} + 124 \cdot 117^{10} + 28 \cdot 130^{10} =$

596252685093703386112+782332233396296613888+198914811490638514319 49+596046720266353770836 76+386003777177200000000000

= 119475115811909228515625

Product right terms

$88745 \cdot 65^{10}$ = 119475115811909228515625

Similarly we have many solutions of this equation

Solution

$4223737 \cdot t^{10} + 5412 \cdot u^{10} + 2443549 \cdot v^{10} + 124 \cdot x^{10} + 28 \cdot y^{10} = 88745 \cdot z^{10}$

a = 4223737, t = 26

b = 5412, u = 52

c = 2443549, v = 39

d = 124, x = 117

e = 28, y = 130

f = 88745 z = 65

~~~~~/////~~~~~
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Exercises

1**) Write equation which has the form below

$$a\cdot v^3 + b\cdot x^3 + c\cdot y^3 = d\cdot z^3$$

Find the value of variables of this equation

Give zeta function ζ(s) is equal to 1/3 and ζ(1/3) is equal to 0

2**) Write equation which has the form below

$$a\cdot v^3 + b\cdot x^3 + c\cdot y^3 = d\cdot z^3$$

Find the value of variables of this equation

Give zeta function ζ(s) is equal to 407/27 and ζ(407/27) is equal to 0

3**) Write equation which has the form below

$$a\cdot v^3 + b\cdot x^3 + c\cdot y^3 = d\cdot z^3$$

Find the value of variables of this equation

Give zeta function ζ(s) is equal to 7/27 and ζ(7/27) is equal to 0

4**) Write equation which has the form below

$$a\cdot v^3 + b\cdot x^3 + c\cdot y^3 = d\cdot z^3$$

Find the value of variables of this equation

Give zeta function ζ(s) is equal to 13/27 and ζ(13/27) is equal to 0

5**) Write equation which has the form below

$$a\cdot v^4 + b\cdot x^4 + c\cdot y^4 = d\cdot z^4$$

Find the value of variables of this equation

Give zeta function ζ(s) is equal to 95/81 and ζ(95/81) is equal to 0

6**) Write equation which has the form below

$$a\cdot v^4 + b\cdot x^4 + c\cdot y^4 = d\cdot z^4$$

Find the value of variables of this equation

Give zeta function ζ(s) is equal to 61/9 and ζ(61/9) is equal to 0

7**) Write equation which has the form below

$$a\cdot v^4 + b\cdot x^4 + c\cdot y^4 = d\cdot z^4$$

Find the value of variables of this equation

Give zeta function ζ(s) is equal to 17/81 and ζ(17/81) is equal to 0

8**) Write equation which has the form below

$$a\cdot v^4 + b\cdot x^4 + c\cdot y^4 = d\cdot z^4$$

Find the value of variables of this equation

Give a = 2401

9**) Write equation which has the form below

$$a\cdot v^4 + b\cdot x^4 + c\cdot y^4 = d\cdot z^4$$

Find the value of variables of this equation

Give b = 14641

10**) Write equation which has the form below

$a\cdot v^4 + b\cdot x^4 + c\cdot y^4 = d\cdot z^4$

Find the value of variables of this equation

Give c = 28561

11**) Write equation which has the form below

$a\cdot v^4 + b\cdot x^4 + c\cdot y^4 = d\cdot z^4$

Find the value of variables of this equation

Give d = 8962

12**)- Write equation which has the form below

$a\cdot v^5 + b\cdot x^5 + c\cdot y^5 = d\cdot z^5$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

13**)- Write equation which has the form below

$a\cdot v^5 + b\cdot x^5 + c\cdot y^5 = d\cdot z^5$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0, a = 3125

14**)- Write equation which has the form below

$$a\cdot v^5 + b\cdot x^5 + c\cdot y^5 = d\cdot z^5$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0 b = 1024

15**)- Write equation which has the form below

$$a\cdot v^5 + b\cdot x^5 + c\cdot y^5 = d\cdot z^5$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0, c = 243

16**)- Write equation which has the form below

$$a\cdot v^5 + b\cdot x^5 + c\cdot y^5 = d\cdot z^5$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0, d = 17050

17**)- Write equation which has the form below

$$a\cdot v^6 + b\cdot x^6 + c\cdot y^6 = d\cdot z^6$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

18**)- Write equation which has the form below

$$a\cdot v^7 + b\cdot x^7 + c\cdot y^7 = d\cdot z^7$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

19**)- Write equation which has the form below

$$a\cdot v^5 + b\cdot x^5 + c\cdot y^5 = d\cdot z^5$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to 379/16807 and ζ(379/16807) is equal to 0

20**)- Write equation which has the form below

$$a\cdot v^8 + b\cdot x^8 + c\cdot y^8 = d\cdot z^8$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

20**)- Write equation which has the form below

$$a\cdot v^9 + b\cdot x^9 + c\cdot y^9 = d\cdot z^9$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

21**)- Write equation which has the form below

$$a\cdot v^{10} + b\cdot x^{10} + c\cdot y^{10} = d\cdot z^{10}$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

22**)- Write equation which has the form below

$$a\cdot v^{11} + b\cdot x^{11} + c\cdot y^{11} = d\cdot z^{11}$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

23**)- Write equation which has the form below

$a\cdot u^4 + b\cdot v^4 + c\cdot x^4 + d\cdot y^4 = e\cdot z^4$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

24**)- Write equation which has the form below

$a\cdot u^4 + b\cdot v^4 + c\cdot x^4 + d\cdot y^4 = e\cdot z^4$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0, a = 14641

25**)- Write equation which has the form below

$a\cdot u^5 + b\cdot v^5 + c\cdot x^5 + d\cdot y^5 = e\cdot z^5$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

26**)- Write equation which has the form below

$$a\cdot u^5 + b\cdot v^5 + c\cdot x^5 + d\cdot y^5 = e\cdot z^5$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0, b = 248832

27**)- Write equation which has the form below

$$a\cdot u^6 + b\cdot v^6 + c\cdot x^6 + d\cdot y^6 = e\cdot z^6$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

28**)- Write equation which has the form below

$$a\cdot u^6 + b\cdot v^6 + c\cdot x^6 + d\cdot y^6 = e\cdot z^6$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0, e = 50752

29**)- Write equation which has the form below

$$a\cdot u^7 + b\cdot v^7 + c\cdot x^7 + d\cdot y^7 = e\cdot z^7$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

30**)- Write equation which has the form below

$$a\cdot u^7 + b\cdot v^7 + c\cdot x^7 + d\cdot y^7 = e\cdot z^7$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0, x = 98

31**)- Write equation which has the form below

$$a\cdot u^8 + b\cdot v^8 + c\cdot x^8 + d\cdot y^8 = e\cdot z^8$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

32**)- Write equation which has the form below

$$a\cdot u^9 + b\cdot v^9 + c\cdot x^9 + d\cdot y^9 = e\cdot z^9$$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

33**)- Write equation which has the form below

$a\cdot u^{11} + b\cdot v^{11} + c\cdot x^{11} + d\cdot y^{11} = e\cdot z^{11}$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

34**)- Write equation which has the form below

$a\cdot u^{12} + b\cdot v^{12} + c\cdot x^{12} + d\cdot y^{12} = e\cdot z^{12}$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

35***)- Write equation which has the form below

$a\cdot t^{10} + b\cdot u^{10} + c\cdot v^{10} + d\cdot x^{10} + e\cdot y^{10} = f\cdot z^{10}$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

36***)- Write equation which has the form below

$a\cdot t^{10} + b\cdot u^{10} + c\cdot v^{10} + d\cdot x^{10} + e\cdot y^{10} = f\cdot z^{10}$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0, f = 283523825

37***)- Write equation which has the form below

$a\cdot t^{11} + b\cdot u^{11} + c\cdot v^{11} + d\cdot x^{11} + e\cdot y^{11} = f\cdot z^{11}$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

38***)- Write equation which has the form below

$a\cdot t^{12} + b\cdot u^{12} + c\cdot v^{12} + d\cdot x^{12} + e\cdot y^{12} = f\cdot z^{12}$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

39***)- Write equation which has the form below

$a\cdot t^{13} + b\cdot u^{13} + c\cdot v^{13} + d\cdot x^{13} + e\cdot y^{13} = f\cdot z^{13}$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

40***)- Write equation which has the form below

$a \cdot t^{14} + b \cdot u^{14} + c \cdot v^{14} + d \cdot x^{14} + e \cdot y^{14} = f \cdot z^{14}$

-Find the value of variables of this equation

-Verify for sure

Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0

41***)- Write equation which has the form below

$a \cdot t^{15} + b \cdot u^{15} + c \cdot v^{15} + d \cdot x^{15} + e \cdot y^{15} = f \cdot z^{15}$

-Find the value of variables of this equation

-Verify for sure (Give zeta function ζ(s) is equal to k and ζ(k) is equal to 0)

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About the Author

Ran Van Vo was born in May 1945 in Quang Ngai, Vietnam. Before 1975, he was an officer in the South Vietnam Army during the Civil War.

After 1975, the Communists took over the South Vietnam, and he was placed in jail for three years.

In 1989, he applied for HO -- a refugee program in United States, and he got a reply from the American Embassy saying that they accepted his request.

In 1990, his family had six people,(his wife : Sen Thi Tran, two daughters and two sons) and they have started a new life in Houston, TX, in the United States. A year after living in the United States, he became interested in the World of mathematics; about Fermat's Last Theorem. Thus, in March 2002, he began his quest for knowledge in mathematics.

He is now living in Wichita, KS.